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SPIE PRESS
Bellingham, Washington USA

Library of Congress Cataloging-in-Publication Data

Names: Hyde, Milo W., author.
Title: Computational optical coherence and statistical optics / Milo W. Hyde, IV, Air Force Institute of Technology.
Description: Bellingham, Washington : SPIE–The International Society for Optical Engineering, 2023. | Includes bibliographical references and index.
Identifiers: LCCN 2022037732 | ISBN 9781510657175 (paperback) | ISBN 9781510657168 (pdf)
Subjects: LCSH: Coherence (Optics)–Computer simulations. | Coherence (Optics)–Statistical methods.
Classification: LCC QC476.C6 H93 2023 | DDC 535/.2–dc23/eng20221209
LC record available at https://lccn.loc.gov/2022037732

Published by
SPIE
P.O. Box 10
Bellingham, Washington 98227-0010 USA
Phone: +1 360.676.3290
Fax: +1 360.647.1445
Email: books@spie.org
Web: www.spie.org

Printed in the United States of America.
First printing 2023.
For updates to this book, visit http://spie.org and type “PM356” in the search field.

SPIE.

To Cristina, Elissa, and Anna—my world
To Elissa and Anna, I'm sorry; this book is not about unicorns and fairies.

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The supplemental material for this book is available in Appendix C and for download here: https://spie.org/Samples/Pressbook_Supplemental/PM356_sup.zip

Preface

The field of optics can generally be divided into four subfields or disciplines, namely, geometrical, wave, statistical, and quantum optics. Geometrical or ray optics is by far the oldest and most mature subfield, having been studied since the time of Fermat and Newton. Geometrical optics models light as a ray and is accurate in the asymptotic limit as the wavelength goes to zero. As a result, geometrical optics does not accurately predict phenomena such as diffraction (although it can be extended to include such phenomena via the geometrical theory of diffraction and the uniform theory of diffraction). Wave optics—developed by giants like Fresnel, Young, Maxwell, Rayleigh, and Sommerfeld—includes diffraction, interference, and all other wave phenomena and is the second most mature discipline. The most popular application of wave optics theory is Fourier optics, so much so that the two are now synonymous. Both geometrical and wave optics are extensively used in optical design and have been the subject of numerous theoretical and computational textbooks.

Statistical optics, as it is commonly defined, extends both geometrical and wave optics to include random optical sources, propagation through or scattering from random media, and detector noise. Major contributors to the discipline include Wolf, Goodman, Tatarskii, and Ishimaru. Indeed, Wolf (co-authored with Mandel) and Goodman, respectively, are the authors of what are universally considered the definitive texts on the subject: *Optical Coherence and Quantum Optics* and *Statistical Optics*, now in its second edition. These books present the theoretical foundations of statistical optics and classical optical coherence in excellent physical detail.

Quantum optics arose as a discipline around the time of the first lasers in the 1960s. It includes all aspects of geometrical, wave, and statistical optics and accurately predicts the interaction of individual photons with atoms, the inner workings of lasers, squeezed light states, photoelectric detection, etc. Significant contributors to quantum optics include some of the most brilliant minds in physics—Einstein, Schrödinger, Bohr, Heisenberg, Born, and Mandel. Applications that employ statistical and quantum optics theory are legion: in the case of the former, adaptive optics, optical communications, optical tweezing, directed energy, and remote sensing, and in the latter, lasers, quantum communications, and quantum computing. This list is by no means all-inclusive.

While some of the technologies and applications listed above are well established, many are still in development, and consequently, active areas of research. Statistical optics is a little more mature in this regard. In recent years, statistical optics and classical optical coherence theory have been applied to engineer and synthesize random fields for use in specific applications, many of which are mentioned above. Indeed, techniques to physically generate optical fields with prescribed correlation or coherence properties can be found throughout the published literature. Two recent papers in *Progress in Optics* entitled "Generation of partially coherent beams" (*Prog. Opt.* 2017, **62**, 157–223) and "Applications of optical coherence theory" (*Prog. Opt.* 2020, **65**, 43–104) provide excellent summaries of these topics. As novel technologies and applications increasingly exploit optical coherence, accurate simulation of stochastic optical fields becomes critically important. Recent books on statistical optics have started to include sections on simulating random optical fields. Nevertheless, unlike geometrical and wave optics, currently there is no text (to my knowledge) devoted to this topic.

This book aims to be the first by presenting current approaches for simulating random optical fields with prescribed statistical properties. In particular, this text demonstrates how to generate optical fields, which are sample functions drawn from a random process described by a correlation function. These random fields can then be used in simulations of optical systems, propagation through random or complex media, scattering from surfaces, etc., which are described in other computational optics texts, like *Computational Fourier Optics* (SPIE Press, 2011), *Optics Using MATLAB®* (SPIE Press, 2017), *Numerical Simulation of Optical Wave Propagation* (SPIE Press, 2010), *Computational Methods for Electromagnetic and Optical Systems* (CRC Press, 2011), and *Computational Photonics* (Wiley, 2010).

The secondary purpose of this book is as a teaching tool, augmenting the theoretical concepts presented in Wolf's and Goodman's classic texts. Traditionally, students of optics begin with geometrical and wave optics, which are taught assuming deterministic optical fields. The transition to the concept of a random optical field can be difficult to grasp, especially when the mathematics requires understanding and applying random process theory. On the other hand, by generating realizations of the random optical field, the statistical optics problem simplifies to a deterministic geometrical or wave optics problem with which students are more familiar. In the context of statistical optics, this simulation is a single random experiment, and statistical moments are computed from the outcomes of many such independent experiments. It has been my experience that this Monte Carlo approach to statistical optics provides a significant amount of insight into the underlying physical phenomena, which greatly exceeds that from theory alone.

This book is intended for senior undergraduate- and graduate-level students studying optical physics and engineering as well as researchers or engineers working in optics. The topics covered in this text require a working knowledge of differential and integral calculus, probability and statistics, random processes, linear systems, and MATLAB® programming. It is impossible to include all the background information on a topic as broad as statistical optics and keep the text at a manageable length. Therefore, this book includes extensive reference lists where many of these details can be found.

This textbook is organized into six chapters and three appendices. Chapter 1 briefly reviews scalar diffraction theory—including the plane wave spectrum, Rayleigh–Sommerfeld, Fresnel, and Fraunhofer diffraction—before discussing the foundational principles of scalar statistical optics. We begin with the first-order or single-point statistics of polarized thermal and pseudo-thermal light, presenting the probability density functions (PDFs) and statistical moments of the instantaneous field and irradiance. We then proceed to second-order (two-point) statistics of the optical field and review key concepts such as the mutual coherence function, cross-spectral density (CSD) function, the coherent-modes representation of the CSD function, the superposition rule, and the van Cittert–Zernike theorem. We close the chapter with a review of second-order irradiance statistics of thermal light sources, including the covariance of irradiance, integrated irradiance, and intensity interferometry also known as the Hanbury Brown and Twiss effect.

In Chapter 2, we present several methods for generating random scalar fields given a CSD function. These simulation techniques include coherent modes, pseudo-modes, and the superposition rule. Step-by-step instructions are provided for implementing each of these techniques, and we generate multiple random sources using these algorithms. All of the MATLAB scripts are explained in detail prior to analyzing the results, and the source code is provided in Appendix C and electronically as part of this book (see supplemental material 🔗).

Chapter 3 generalizes the theory presented in Chapter 1 to vector or electromagnetic random fields. In this chapter, we begin by reviewing vector diffraction theory, the polarization ellipse, Jones vectors, Stokes parameters, and the Poincaré sphere. We then proceed to the first-order statistics of partially polarized thermal light and discuss such concepts as the coherency matrix, the degree of polarization, the polarization state of random fields, and the PDFs of the Stokes parameters. This is followed, quite naturally, by a review of the second-order moments of the optical field. The topics presented here are the beam coherence-polarization matrix (BCPM), the CSD matrix (CSDM), the electromagnetic coherent-modes representation, bimodal expansions of the CSDM, and the electromagnetic superposition rule. Lastly, we conclude the chapter with a brief summary of second-order irradiance statistics of partially polarized thermal light sources.

Chapter 4 discusses several methods for generating random electromagnetic fields, including bimodal expansions, vector pseudo-modes, and the electromagnetic superposition rule. Like in Chapter 2, step-by-step instructions are provided for implementing each of these techniques, and we generate example electromagnetic random sources using these algorithms. All of the MATLAB scripts are explained in detail.

In Chapter 5, we apply the concepts and algorithms from the prior chapters to analyze and simulate classical statistical optics experiments and instruments, as well as applications that utilize random light. Included in this chapter are detailed simulations of the double-slit or Young's experiment, a Michelson interferometer, beam and polarization control with stochastic fields, the Hanbury Brown and Twiss experiment, and imaging with partially coherent light.

Chapter 6 describes how to simulate nonstationary or pulsed random fields. Nonstationary partially coherent sources, especially those with space-time or spatiotemporal coupling, have recently gained interest for potential use in optical trapping, optical tweezing, and atomic optics. They are currently at the forefront of beam-control research. What makes simulating nonstationary random fields especially interesting is the ability to observe the time evolution of the source. This can provide significant insight into how random fields behave. We begin this chapter with a summary of the germane theory—including reviews of the BCPM, coherent modes and bimodal expansions of the BCPM, pseudo-modes, and the superposition rule—before generating three example thermal, nonstationary sources. As part of these simulations, we create movies showing the temporal evolution of these random fields, which are included with the MATLAB code that accompanies this book. We discuss the fields' physical behaviors in the text.

Lastly, besides the MATLAB source code in Appendix C, the appendices cover two topics that are generally useful when simulating optical propagation, be it deterministic or random. The first, in Appendix A, explains how to simulate wave propagation through optical systems (described by a ray-tracing **ABCD** matrix) by evaluating the Collins formula, also known as the generalized Huygens–Fresnel integral, using fast Fourier transforms. In the appendix, we derive the sampling constraints for two forms (specifically, the Fourier transform and convolution form) of the Collins formula and present an example where we simulate wave propagation through an astigmatic optical system. In Appendix B, we describe how to simulate fields with high spatial frequency content (spatially broadband fields) via Fresnel spatial filtering. Fields of this type include point sources (deterministic) and spatially incoherent fields (stochastic). We first present the theory underpinning Fresnel spatial filtering and then apply the technique to simulate propagation of a spatially incoherent source.

It has become somewhat of a cliché but is nonetheless true: No one writes a book alone. There are many people that deserve my thanks for making it possible. First, I would like to acknowledge my Master's research advisor Prof. Michael Havrilla. His insistence on linking the mathematics to physical understanding has motivated all of my work in electromagnetics and optics. Second, I would like to thank my doctoral advisor Dr. Jason Schmidt. He is the most knowledgeable person in numerical wave propagation that I know, and his lessons on the subject heavily influenced this work. These two individuals are the most responsible for giving me the knowledge to write this book, and I am eternally grateful.

Other people that played major roles in this effort are Dr. Santasri Bose-Pillai, Dr. Jack McCrae, and Prof. Steven Fiorino at the Center for Directed Energy of the Air Force Institute of Technology (AFIT) and Dr. Mark Spencer at the Directed Energy Directorate of the Air Force Research Laboratory (AFRL). The latter two have been extremely generous providing financial support for my research. They made many of the simulation topics covered in this book possible. I would also like to thank Prof. David Voelz at New Mexico State University and Prof. Olga Korotkova at the University of Miami for many fruitful research collaborations. I look forward to many more in the future.

Last and certainly not least, I am incredibly grateful for my family—Cristina, Elissa, and Anna. Your patience and understanding while I spent seven days a week for nine months writing this book have been incredible.

Milo Hyde
3 May 2022

Acronyms

BCPM	beam coherence-polarization matrix
BGCSM	Bessel-Gaussian correlated Schell-model
CCG	circular complex Gaussian
CDF	cumulative distribution function
CDoC	complex degree of coherence
CSD	cross-spectral density
CSDM	cross-spectral density matrix
DoP	degree of polarization
EGPSM	electromagnetic Gaussian pseudo-Schell-model
EGSM	electromagnetic Gaussian Schell-model
EHGCSM	electromagnetic Hermite-Gaussian correlated Schell-model
EMGSM	electromagnetic multi-Gaussian Schell-model
FFT	fast Fourier transform
GSM	Gaussian Schell-model
HBT	Hanbury Brown and Twiss
MCF	mutual coherence function
MGSM	multi-Gaussian Schell-model
NUC	non-uniformly correlated
PM	pseudo-modes
PMF	probability mass function
PDF	probability density function
PSF	point spread function
SDoC	spectral degree of coherence
SM	Schell-model
TEM	transverse electromagnetic
VCZT	van Cittert–Zernike theorem
WSS	wide-sense stationary