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COMMUNICATION

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Abstract. The polarizability of an arbitrarily shaped electrically small particle of an isotropic dielectric material embedded in an isotropic dielectric host material given in a recent article [*Opt. Eng.* **52**, 051205 (2013)] is incorrect. In particular, the polarizability cannot be a scalar (unless the particle is either spherical or cubical), but must be a dyadic. [DOI: 10.1117/1.OE.52.7.079701]

In a recent article, La Spada et al.[1] considered an electrically small particle made of an isotropic dielectric material of permittivity ϵ_i embedded in an isotropic host material of permittivity ϵ_e. It was stated in Ref. 1 that "that the polarizability of an arbitrary shaped particle can be expressed as

$$\alpha = V\epsilon_e \frac{\epsilon_i - \epsilon_e}{\epsilon_e + L(\epsilon_i - \epsilon_e)}, \tag{1}$$

where V is the particle volume ... and L is the depolarization factor." Clearly, the polarizability α is a scalar in this equation. Equation (2) in Ref. 1 further reinforces the scalar natures of the depolarization factor and the polarizability without any preconditions on the shape of the particle and the orientation of an electric field incident on it.

However, the reader should note that the polarizability of an electrically small particle of an isotropic dielectric material embedded in an isotropic host material cannot be a scalar unless the particle is either spherical or cubical. The polarizability of an arbitrarily shaped particle has to be a dyadic (or, equivalently, a second-rank tensor), because depolarization cannot be represented by a scalar L. Therefore, Eq. (1) in Ref. 1 does not have the correct structure for an electrically small particle of arbitrary shape.

The foregoing is illustrated by the polarizability of an ellipsoidal particle being correctly represented as the dyadic

$$\underline{\underline{\alpha}} = V\epsilon_e \sum_{\ell=1}^{3} \frac{\epsilon_i - \epsilon_e}{\epsilon_e + L_\ell(\epsilon_i - \epsilon_e)} \underline{u}_\ell\, \underline{u}_\ell, \tag{2}$$

where $\underline{u}_\ell$ are unit vectors in the directions of the principal axes of the ellipsoidal particle and L_ℓ are the three components of the corresponding depolarization dyadic $\underline{\underline{L}} = L_1 \underline{u}_1\, \underline{u}_1 + L_2 \underline{u}_2 \underline{u}_2 + L_3 \underline{u}_3 \underline{u}_3$. For prolate and oblate spheroidal shapes, expressions for $\underline{\underline{\alpha}}$ and L_ℓ were provided by David in 1939.[2] For more general ellipsoidal shapes, expressions for L_ℓ in terms of elliptic functions were published in 1945 by Osborn[3] and Stoner.[4] In fact, integral equation-based formulations of the polarizability dyadic are now available for the general case of a bianisotropic ellipsoidal particle embedded in a bianisotropic host material[5] based on the corresponding depolarization dyadic.[6]

La Spada et al. also derived expressions for the absorption and scattering cross-sections of an ellipsoidal particle, after explicitly confining themselves to the case of the incident electric field being parallel to the longest of the three principal axes of the particle. In that special case, one diagonal component of the polarizability dyadic alone is indeed sufficient, but that specialization cannot reduce the polarizability dyadic to a scalar, because the polarizability dyadic does not depend on the direction of the incident electric field.

References

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