

Mapping and violating Bell inequality with entangled photons

David A. Guzmán*, Leonardo J. Uribe, Alejandra Valencia, Ferney J. Rodríguez, Luis Quiroga
Physics Department, Universidad de los Andes, A.A. 4976, Bogotá D.C., Colombia

ABSTRACT

In 1964, J. Bell introduced an inequality that stated a mathematical bound for any physical system that holds both locality and realism; if we violate this inequality, it is clear that we have to reconsider the previous statement. In our work, we report an experimental activity with photons suitable for undergraduate students that makes them question these naïve ideas of nature's behavior.

With a pre-aligned setup, our students tested and violated Bell's inequality in a two-hour laboratory session, using two distant photons entangled in polarization. In addition, complementing an educational approach to this phenomenon, the usually called S function, that quantifies correlations, was mapped using different detection angles in one of the two locations. In particular, a more complete picture of the S function, allow us to identify the initial state of light.

We show in this work that it is possible for undergraduate students to question some of our common sense ideas of nature using experiments with photons.

Keywords: Photons, locality, realism, Bell's inequalities

1. INTRODUCTION

The central issue addressed by Bell's work¹ is about the correlation of results in spatially separated measurements. As clearly stated by Eberly,² and most recently by Zeilinger,³ Bell inequalities do not derive from principles of quantum mechanics. They simply express an upper bound that a certain combination of correlation results should satisfy provided that our common sense ideas of realism and locality hold. The fact that there are systems violating Bell inequalities tell us that nature does not behave according to our everyday experience.

Here, we report an extension of previous educational works^{4,5} that we believe is useful at the moment of illustrating the physical ideas behind Bell inequalities to the students. Our idea consists in measuring the S function for various experimental parameters in sharp contrast with previous experiments in which only a single value of the S function, the one where the Clauser-Horne-Shimony-Holt (CHSH) inequality⁶ is maximally violated, is measured. A particular characteristic of the experiment here reported is the fact that students themselves took the data working in pre-aligned setups in a two-hour laboratory session. This activity was part of a quantum optics course taken by both last year undergraduate and first year graduate students.

2. EXPERIMENTAL ACTIVITY WITH THE STUDENTS

The objective of the practice was to show the students that indeed, in nature, there are physical systems that violate an inequality of the type CHSH indicating that local realism has to be reconsidered. Photon's polarization was chosen as the variable to measure the different outcomes, look at their correlations and find a value of S, which under a local realism theory is limited by

$$|S| \leq 2. \quad (1)$$

In particular, paired photons in the Bell state $|\Psi^-\rangle = (|HV\rangle - |VH\rangle)/\sqrt{2}$ produced by spontaneous parametric down conversion (SPDC) were used as the physical system. Nowadays, the generation of this state is routinely done in different laboratories for research purposes. However, the implementation of an educational activity where undergraduate students could by themselves test locality and realism is not that simple: it requires laboratory sessions that do not fit in the schedule of a typical class and need that the students have certain experimental skills that are not the scope of a quantum mechanics class.

* da.guzman35@uniandes.edu.co

For these reasons for the activity here reported, an instructor implemented a setup, as the one depicted in figure 1, which was later used by pairs of students, in two-hour laboratory sessions. As preparation for the session, students were asked to read an outreach explanation of the phenomena³ to introduce them to the background theory, and one paper about the experimental setup⁴ to introduce them to the technical details.

For our implementation, a 405nm laser with output power of 30mW impinges a nonlinear crystal, a BBO, which has been cut and aligned to produce pairs of photons in the Bell state $|\Psi^-\rangle$ by type II SPDC. It is worthy to mention that our proposal works for any other source of polarization entangled pairs of photons. Each photon from the entangled pair goes through a mirror to pass then by a half waveplate (HWP), which is mounted in a rotational stage. Afterwards each photon's polarization is measured by using a polarizing beam splitter (PBS) and a pair of detectors.

Students measured crossed correlations from coincident events between the polarization of photons for different angle configurations of the used HWPs. First they obtain the probability of measuring a coincident event for photon 1 under polarization X in a basis given by a HWP set at an angle α , and photon 2 under polarization Y in a basis given by its corresponding HWP set at an angle β ; this probability is labeled as $P_{XY}(\alpha, \beta)$.

From these probabilities, a correlation function $E(\alpha, \beta)$ is defined as

$$E(\alpha, \beta) = P_{HH}(\alpha, \beta) + P_{VV}(\alpha, \beta) - P_{HV}(\alpha, \beta) - P_{VH}(\alpha, \beta), \quad (2)$$

which is a quantity measured live during the session for a particular setting of HWPs, i.e. α and β . The goal of this practice is to find values of S such that equation (1) is violated. Function S is defined in terms of correlations in equation (2) given at two different settings of α and β , as

$$S(a, a', b, b') = E(a, b) - E(a, b') + E(a', b) + E(a', b'). \quad (3)$$

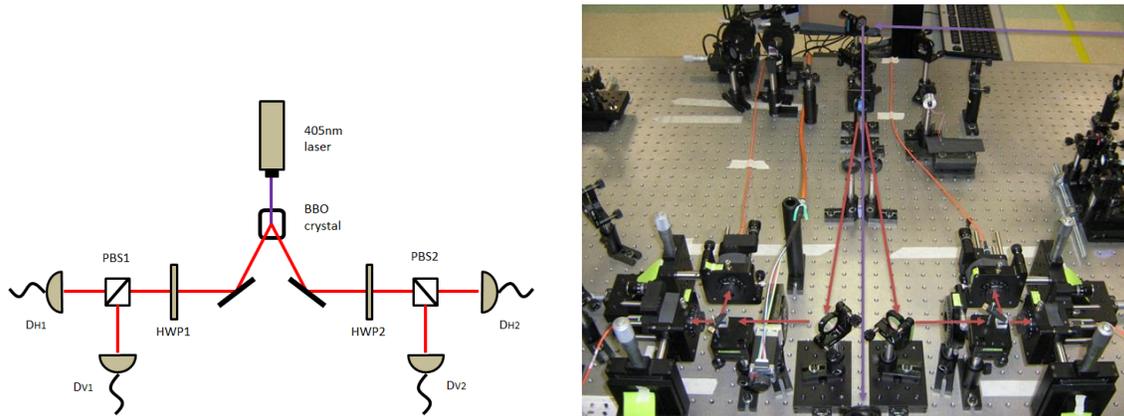


Figure 1. Experimental setup for mapping the S function with entangled photons. Red lines depict the trajectory of the entangled photons. Left: schematic. Right: photograph of the actual setup; laser is outside the picture.

Laboratory session

Quantum mechanics predicts that for the $|\Psi^-\rangle$ Bell state, it can be found a minimum evaluation of $S = -2\sqrt{2}$ at $a=0^\circ$, $a'=45^\circ$, $b=22.5^\circ$ and $b'=67.5^\circ$. This in terms of experimental settings corresponds to $HWP_1=\{0^\circ, 22.5^\circ\}$ and $HWP_2=\{11.25^\circ, 33.75^\circ\}$.

A mapping of the S function is done by students, first by fixing $a=0^\circ$ and scanning b, and then by fixing $a'=45^\circ$ and scanning b as well. For our implementation we obtained a number of total coincidences of $N=108\text{cps}$ (counts per second), students scanned the second HWP for 13 angles, and collected samples for 150s at each position. Including the time that took to rotate the waveplates, each scan lasted about 40 minutes, making possible to run the whole experiment without pressure under a 2 hour session.

Students' results

After the laboratory session, students were asked to hand in a report which includes the theoretical calculation of the S function evaluated at the parameters used in the experimental session, i.e. $S(0^\circ, 45^\circ, b, b')$, for the $|\Psi^-\rangle$ Bell state. They were asked to contrast their experimental measured values for the mapping of S with theory. They should determine if their results violate equation (1), and quantify this violation. Finally students might discuss on the realism and non-locality of the experiment.

Each group reported their experimental results of the mapping of S by scanning variables b and b' . An example of obtained results is depicted in Figure 2. Main results obtained with the setup are listed on Table 1.

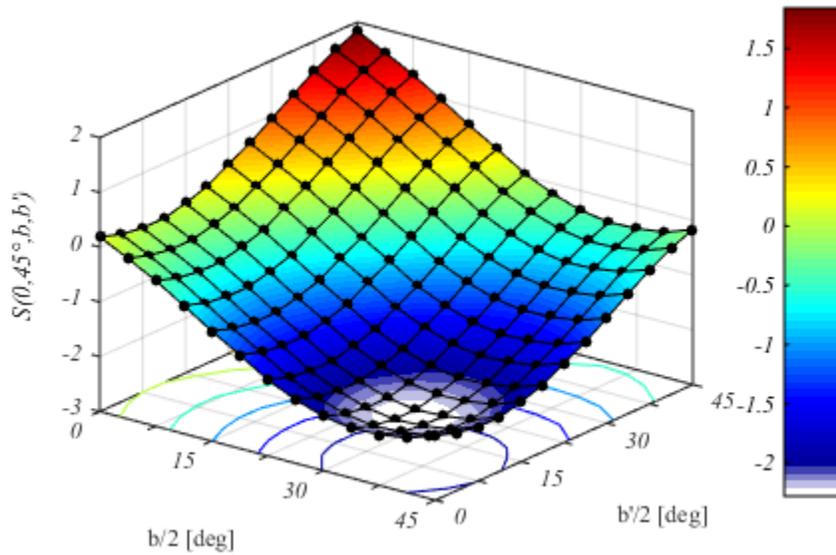


Figure 2. Experimental results for the mapping of the S function by scanning angles of b . Marked points on grid represent experimental values for S obtained from the setup used in the practice. White region on the bottom of the surface shows values where $S < -2$, therefore violating Bell inequality, where several points lie.

The fact that one photon performs measurements for different HWP angles, preparing different projection states, allows observing the behavior of the S function, something that is not possible in studies that report just the value of S that violates optimally equation (1). This approach presents two advantages: First, it constitutes a method of state discrimination, and second, a method to measure if there is or not a violation of Bell inequality without the necessity of knowing the input state.

Table 1. Experimental results reported by students for the maximal measured value of S . Each of them violates equation (1).

Group	$\max(S(0^\circ, 45^\circ, b, b'))$	b and b' for $\max(S)$
Group 1	2.15 ± 0.09	$b=22.5^\circ, b'=82.5^\circ$
Group 2	2.15 ± 0.07	$b=22.5^\circ, b'=67.5^\circ$
Group 3	2.17 ± 0.07	$b=30^\circ, b'=75.0^\circ$
Instructor	2.283 ± 0.023	$b=22.5^\circ, b'=67.5^\circ$
Theory	2.828	$b=22.5^\circ, b'=67.5^\circ$

3. CONCLUSIONS

We report an experimental practice in which students are exposed to physical systems that motivates them to question and contrast their common sense ideas of locality and realism. Differently from other reports on this topic, the students reconstructed the CHSH-S function for different type of measurements getting an idea of its behavior and illustrating that a state can violate Bell inequality for some experimental settings and for others not. Additionally, the map of the CHSH-S function allows looking for violations of Bell inequality independently of the entangled state produced by the source.

The reported experiments were performed by last year undergraduate and first year graduate students using pairs of photons generated via SPDC and considering the polarization state $|\Psi^-\rangle = (|HV\rangle - |VH\rangle)/\sqrt{2}$. The measurements were done in an activity that lasted 2 hours working in pre-aligned setups using readily available optical elements found in a quantum optics laboratory. In the laboratory reports handed by the students, it was clear from their conclusions that they found an incompatibility between their classical conceptions of realism and locality and the results obtained in their experiments. Some of them commented on the necessity of developing a new physical theory in order to explain the behavior of photons in the laboratory. This demonstrates that the experimental practice led them to question their macroscopic common sense ideas. We consider that the present work contributes to interesting possibilities for undergraduate courses such as quantum mechanics and quantum optics.

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