Nonlinear plasmonics with Kerr-like media for sensing

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ABSTRACT

Sensing technologies are currently needed for better maintainability, reliability, safety, and monitoring small variable changes on microscopic and nanoscale systems. Plasmonic sensor research has contributed to chemical and biological sensing needs by monitoring ultrafast temporal and spatial changes in optoelectronic systems. Nonlinear plasmonic waveguides with subwavelength confinement can further enhance the capabilities of plasmonic devices. Results in this paper highlight the derivation of the full-vector Maxwell Equations for the single metal- dielectric slot waveguide and the metal –dielectric –metal waveguide with the dielectric having a Kerr-like nonlinearity. These waveguides, typically have metallic losses that compete with nonlinearity at certain frequencies that can hinder surface plasmon wave propagation. By considering temporal and spatial beam propagation in these waveguides one expects to observe novel effects that could be used for sensing applications such as femtosecond pulse propagation with plasmon self-focusing, self-trapping, and frequency conversion with reduction in metallic losses.

Keywords: Nonlinear plasmonic waveguides, Nonlinear Kerr Effect, nonlinear surface waves, surface plasmons, optical bistability, solitary waves

1. INTRODUCTION

There is a current need for communications systems to be smaller, faster, increased bandwidth, and with more robustness. There are fundamental limitations on electronic and optical technologies such as material fabrication and diffraction effects. Nanotechnology research can address these issues in particular plasmonics. Optical fields coupled to electron oscillations that are limited at a metal/dielectric interface are called surface plasmons (SPs). These fields are squeezed light that are confined to the subwavelength. The SPs exist in other structures besides waveguides such as triangular grooves, slot waveguides, spheres, cones, and arrays[3,5,22,25]. SPs contribute to electric field enhanced in Surface Enhanced Raman Scattering (SERS) [3]. The current research can lead to development of SP based components, devices, and circuits such as waveguides, surface enchanced Raman scattering (SERS) sensors, nanoantennas, resonator structures, integrated platform electronic/optical structures. Even soliton propagation is approximated based on plasmonic based equation scalar models such as the NLSE (or Nonlinear Schordinger Equation). Spatial soliton propagation is established by opposing phenonmena Kerr or Kerr-like effect (self focusing or self trapping) and diffraction. These solitons also exist below the diffraction limit [30-32] or subwavelength spatial solitons at the dielectric\metallic interface. Now, with the well known features of plasmonic waveguides with dielectric\metallic interface, can waveguides propagate with a Kerr or Kerr-like layer (or nonlinear medium)? It turns out that surface waves or SPs exist for dielectric\metallic\nonlinear medium. Recently, graphene layers or thin layers have been an area of interest for researchers in waveguides due to its similiarities to metal properties. Although SPs do not exist for the TE mode in a dielectic\metallic waveguide these surface waves can exist with graphene waveguides. Graphene layers that are very thin compared to incident wavelength can be approximated to boundary conditions in Maxwell's Equations to caluculate dispersion relations and transmission/reflection coefficients[4,21,23].

In this work, surface waveguide physics for dielectric\nonlinear interface is briefly presented to solve analytical solutions to nonlinear equations for tranverse electric(TE) and transverse magnetic (TM) with conductivity considering a thin graphene layer in the boundary conditions. Then, using the first integral approach, a dispersion relation is calculated for the TM mode with a derived equation for the power in the TM mode. Lastly, the single layer dielectric\thin graphene\nonlinear medium and the multilayered (dielectric\thin graphene\dielectric\thin graphene\nonlinear medium) waveguide are studied to calculate reflection and transmission coefficients but a different approach is taken for the intensity dependent index of refraction not normally encountered coefficient calculations.

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2. NONLINEAR SURFACE WAVES IN WAVEGUIDES

Waveguides have been studied typically in nanostructures such as dielectric\metallic, Kerr media\dielectric, dielectric\metallic\dielectric, metallic\dielectric\metallic or, recently dielectric/graphene interface. In a Kerr medium the change in the index of refraction or dielectric constant is driven by the electric field intensity source for isotropic medium and 2 or 1 dimensional electric field components for an anisotropic medium. Kerr-like media has been long associated with polarization field enhancement in nonlinear optics. This nonlinearity could be useful in field enhancement in plasmonics. The nonlinear media contributes to creation of temporal or spatial soliton propagation [1,27]. The models are approximated from Maxwell's or Helmholtz vector equations to a scalar equation called the Nonlinear Schrödinger Equation (NLSE). In the Kerr medium, the coefficient of the intensity of the electric field has to be considered self-focusing or defocusing. The metallic/Kerr-like and dielectric/Kerr-like have been solved previously by G.I. Stegeman, J. Ariyasu, K. M. Leung, J.J. Burke, Jian-Guo Ma, Liu, Bing-Can, and others [7-14, 17, 18]. Recently, graphene has been material considered to behave similar to metal at certain temperatures, chemical potential, electron energy and incident wavelengths that can support creation of SPs at not only TM modes but also TE modes unlike metals cannot support creation SPs [16, 25, and 26].

2.1 Theory

The initial step of the derivation is to solve for surface waves in a semi-infinite wave guide with a Kerr/graphene/dielectric interface. Starting with Maxwell's equations, the next step to is to solve for the TM (or Transverse Electric) modes of the fields. Also, steps were taken to solve for the TE (or Transverse Electric) modes knowing the plasmonic waves also exist for similar to metal/dielectric interface due to the graphene layer. This layer is much thinner compared to the thickness of the nonlinear Kerr and dielectric media and can be approximated to boundary conditions including surface current density. Using the first integral approach , steps were taken to derive the dispersion relation and calculate the energy flux. In order to describe Kerr-like media propagation it is assumed the normalized Maxwell's Equations in the frequency domain to

$$\nabla \times \vec{E} = k_0 \overline{\overline{\varepsilon}} \vec{\tilde{H}} \tag{1}$$

$$\nabla \times \vec{\tilde{H}} = k_0 \overline{\overline{\varepsilon}} \vec{E} - i\mu_0 c \overline{\overline{\sigma}} \vec{E} \tag{2}$$

$$\nabla \cdot \overline{\overline{\varepsilon}} \vec{E} = \vec{\rho} \tag{3}$$

$$\text{with } \vec{\tilde{H}} = -i\sqrt{\frac{\mu_0}{\varepsilon_0}}\vec{H}, \; k_0 = {}^{2\pi}\!/_{\lambda}, \tag{4}$$

$$\vec{E} = E(x,y,z)e^{i(\beta x - \omega t)}, \text{ and } \vec{H} = H(x,y,z)e^{i(\beta x - \omega t)}.$$

The constants μ_0, $\overline{\overline{\varepsilon}}$, ε_0, and $\overline{\overline{\sigma}}$ are the magnetic permeability, relative permittivity tensor, electric conductivity tensor. Expanding Maxwell's equations into scalar equations and with $\partial_\eta = {}^{\partial}\!/_{\partial\eta}, \eta = x, y, z$ gives

$$\partial_y E_z - \partial_z E_y = \left(\mu_{xx}\tilde{H}_x + \mu_{xy}\tilde{H}_y + \mu_{xz}\tilde{H}_z\right) \tag{5a}$$

$$\partial_z E_x - \partial_x E_z = \left(\mu_{yx}\tilde{H}_x + \mu_{yy}\tilde{H}_y + \mu_{yz}\tilde{H}_z\right) \tag{5b}$$

$$\partial_x E_y - \partial_y E_x = \left(\mu_{zx}\tilde{H}_x + \mu_{zy}\tilde{H}_y + \mu_{zz}\tilde{H}_z\right) \tag{5c}$$

$$\partial_y \widetilde{H}_z - \partial_z \widetilde{H}_y = (\varepsilon_{xx} + \sigma'_{xx}) E_x + (\varepsilon_{xy} + \sigma'_{xy}) E_y + (\varepsilon_{xz} + \sigma'_{xz}) E_z \tag{5d}$$

$$\partial_z \widetilde{H}_x - \partial_x \widetilde{H}_z = (\varepsilon_{yx} + \sigma'_{yx}) E_x + (\varepsilon_{yy} + \sigma'_{yy}) E_y + (\varepsilon_{yz} + \sigma'_{yz}) E_z \tag{5e}$$

$$\partial_x \widetilde{H}_y - \partial_y \widetilde{H}_x = (\varepsilon_{zx} + \sigma'_{zx}) E_x + (\varepsilon_{zy} + \sigma'_{zy}) E_y + (\varepsilon_{zz} + \sigma'_{zz}) E_z \tag{5f}$$

with $\overline{\overline{\sigma'}} = -\frac{i\mu_0 c}{k_0}\overline{\overline{\sigma}}$, $x' = k_0 x, y' = k_0 y$, and $z' = k_0 z$. With the normalized Maxwell equations, we take the prime away from the spatial for convenience ($x' \to x$, $y' \to y$ $z' \to z$). We consider solving for the TM mode of the fields due to the fact plasmonic propagation existence found in this mode. Also, we consider propagation along the x axis so with $\partial_x = \partial/\partial x \to i\beta$ and the TM mode meaning $\partial_y = 0$ and

$\widetilde{H}_x = \widetilde{H}_z = E_y = 0$ so

$$-\partial_z \widetilde{H}_y = (\varepsilon_{xx} + \sigma'_{xx}) E_x \tag{6}$$

$$i\beta \widetilde{H}_y = (\varepsilon_{zz} + \sigma'_{zz}) E_z \tag{7}$$

$$\partial_z E_x - i\beta E_z = \mu_{yy} \widetilde{H}_y. \tag{8}$$

Here, since magnetic material is not considered, the relative permeability $\mu_{yy} = 1$. In the above equations the relative permittivity ε_{xx} in the medium has an Kerr effect or

$$\varepsilon_{xx} = \varepsilon_{0x} \pm \alpha |E_x|^2. \tag{9}$$

The constant ε_{0x} is the permittivity of the Kerr medium and α is the coefficient of the electric field intensity. The purpose of the constant +α (-α) is for focusing (defocusing) Kerr medium. Equation (9) is the uniaxial approximation [1]. The dielectric variable ε_{zz} is approximated $\varepsilon_{zz} \approx \varepsilon_{0x}$ [1,7-14]. We assume no conductivity z direction or σ'_{zz}=0, but assume $\sigma'_{xx} \neq 0$. A more complete form of equations (6-8) is

$$E_x = -\frac{1}{(\varepsilon_{xx} + \sigma'_{xx})} \partial_z \widetilde{H}_y \tag{10}$$

$$E_z = i\frac{\beta}{\varepsilon_{0x}} \widetilde{H}_y \tag{11}$$

$$\partial_z E_x - i\beta E_z = \widetilde{H}_y \text{ or } \widetilde{H}_y = \frac{\varepsilon_{xx}}{(\varepsilon_{xx} - \beta^2)} \partial_z E_x. \tag{12}$$

These equations can also be combined into a scalar wave equation along with equation (9)

$$\partial_z^2 E_x - \gamma N^2 E_x - \alpha N^2 \left|E_x\right|^2 E_x = 0 \text{ with } \partial_\eta^2 = \partial^2 \big/ \partial\eta^2, \eta = x, y, z \text{ and} \tag{13}$$

$$\gamma = \varepsilon_{0x} + \sigma'_{xx} \tag{14}$$

$$N^2 = \frac{\beta^2}{\varepsilon_{0x}} - 1. \tag{15}$$

Equation (13) is the scalar equation with a Kerr effect with focusing. A proposed ansatz for the solution

$$E_x = A \frac{1}{\cosh^\lambda (B(\bar{z} - z))} \tag{16}$$

The trigonometry identity of this function has terms such that as $z \to \infty, E_x \to 0, \partial_z E_z \to 0$. Next, substitute equation (16) into equation (13) which results are

$$-B^2 \lambda \frac{1}{\cosh^2 (B(\bar{z} - z))} + B^2 \lambda^2 - B^2 \lambda^2 \frac{1}{\cosh^2 (B(\bar{z} - z))} - \gamma N^2 - \alpha N^2 \frac{A^2}{\cosh^{2\lambda} (B(\bar{z} - z))} = 0. \tag{17}$$

The conditions to satisfy equation (17) with λ=1 are

$$B = \sqrt{\gamma N^2} \tag{18}$$

$$A = \sqrt{\frac{-2\alpha}{\gamma}} \tag{19}$$

$$E_x = \pm \sqrt{\frac{-2\alpha}{\gamma}} \frac{1}{\cosh(\sqrt{\gamma N^2} (\bar{z} - z))}. \tag{20}$$

The term $\sqrt{\gamma N^2}$ must be greater than zero so for E_x as $z \to \infty$ $E_x(z) \to 0$. It also cannot be purely imaginary. The amplitude of the electric field can be complex and depend on the nonlinear coefficient, dielectric constant, and conductivity. Using equations (10-12), one can solve for the other field components

$$\widetilde{H}_y = \pm \frac{\sqrt{-2\alpha N^2}}{\left(1 - \frac{\beta^2}{\varepsilon_{0x}}\right)} \frac{\tanh(\sqrt{\gamma N^2} (\bar{z} - z))}{\cosh(\sqrt{\gamma N^2} (\bar{z} - z))} \tag{21}$$

$$E_z = \pm i \frac{\beta}{\varepsilon_{0x}} \frac{\sqrt{-2\alpha N^2}}{\left(1-\dfrac{\beta^2}{\varepsilon_{0x}}\right)} \frac{\tanh(\sqrt{\gamma N^2}(\bar{z}-z))}{\cosh(\sqrt{\gamma N^2}(\bar{z}-z))}. \tag{22}$$

Alternatively, the wave equation for defocusing in the TM mode is

$$\partial_z^2 E_x - \gamma N^2 E_x + \alpha N^2 |E_x|^2 E_x = 0 \tag{23}$$

with the solution being

$$E_x = \pm \sqrt{\frac{-2\alpha}{\gamma}} \frac{1}{\sinh(\sqrt{\gamma N^2}(\bar{z}-z))}. \tag{24}$$

We can solve for the TE (or Transverse Electric) modes knowing the plasmonic waves also exist for similar to metal/dielectric interface due to the graphene layer. Again, this layer is much thinner compared to the thickness of the nonlinear Kerr and dielectric media and can be approximated to boundary conditions including surface current density. Also, the existence of TE mode solutions does not necessarily mean existence of plasmons at the interface like a metallic/dielectric interface does not in the TE mode. This depends on graphene conductivity calculation is more intricate than frequency dependent metal. In this mode, $\tilde{H}_y = E_z = E_x = 0$, so the based on equation (5) the electric field E_y wave equation is

$$\partial_z^2 E_y - {N_2}^2 E_y + \alpha |E_y|^2 E_y = 0 \tag{25}$$

with solution being

$$E_y = \pm \sqrt{\frac{2N_2^2}{\gamma}} \frac{1}{\cosh(N_2(\bar{z}-z))} \quad \text{and} \tag{26}$$

$$N_2^2 = \beta^2 - \varepsilon_{0y} - \sigma'_{yy}. \tag{27}$$

The coupled equations in the TE mode are

$$-\partial_z E_y = \tilde{H}_x \tag{28}$$

$$i\beta E_y = \tilde{H}_z \tag{29}$$

$$\partial_z \tilde{H}_x - i\beta \tilde{H}_z = (\varepsilon_{yy} + \sigma'_{yy}) E_y \tag{30}$$

The uniaxial approximation to the nonlinear dielectric material would be

$$\varepsilon_{yy} = \varepsilon_{0y} \pm \alpha \left| E_y \right|^2 . \tag{31}$$

The solutions and the coupled Maxwell relations can be used along with boundary conditions to derive dispersion relations at an interface such a dielectric/very thin graphene/Kerr medium.

2.2 Boundary conditions in the waveguide

If we first consider the TM mode equation (23) taking the first integral (multiplying $\partial_z E_x$ and integrating as function of z) the results are

$$\left(\partial_z E_{x1}\right)^2 - \gamma N^2 \left(E_{x1}\right)^2 - \alpha N^2 \frac{\left(E_{x1}\right)^4}{2} = C. \tag{32}$$

Here, C=0. At z<0, the wave is

$$E_{x2} = E_{x0} e^{qz} \tag{33}$$

$$q^2 = \beta^2 - \varepsilon_s . \tag{34}$$

ε_s is the dielectric constant at z<0. Assuming the graphene is sufficiently thin compared to the Kerr and dielectric media, it is approximated to boundary conditions of normal and tangential components

$$\vec{E}_{1t} = \vec{E}_{2t} \tag{35a}$$

$$\vec{H}_{1t} - \vec{H}_{2t} = \vec{J}_s \times \hat{n} \text{ or } \tilde{\vec{H}}_{1t} - \tilde{\vec{H}}_{2t} = \overline{\overline{\sigma'}}\vec{E} \times \hat{n} \tag{35b}$$

$$D_{1n} - D_{2n} = \rho_s \tag{35c}$$

$$B_{1n} = B_{2n} \tag{35d}$$

which $\vec{J}_s$ and ρ_s is the surface current density and charge density with $\vec{B} = \mu_0 \vec{H}$. $\vec{J}_s \times \hat{n}$ is oriented

$$\sigma_{xx} E_x \hat{x} \times \hat{n} = -\sigma_{xx} E_x \hat{y} \text{ for TM mode and} \tag{36}$$

$$\sigma_{yy} E_y \hat{y} \times \hat{n} = \sigma_{yy} E_y \hat{x} \text{ for the TE mode} \tag{37}$$

recalling $\vec{J}_s = \overline{\overline{\sigma}}\vec{E}$. With equations (35a-d), at z=0 considering the TM mode,

$$E_{x0}\,|_{z=0^-} = E_{x0} = \sqrt{\frac{-2\alpha}{\gamma}}\,\frac{1}{\cosh(\sqrt{\gamma N^2}\,(\bar{z}))} \tag{38}$$

$$i\varepsilon_{nl}\,\frac{\beta}{\varepsilon_{0x}}\sqrt{\frac{2\alpha}{\left(\frac{\beta^2}{\varepsilon_{0x}}-1\right)}}\,\frac{\tanh(\sqrt{\gamma N^2}\,(\bar{z}))}{\cosh(\sqrt{\gamma N^2}\,(\bar{z}))} - \varepsilon_s E_{x0} = \rho_s \tag{39}$$

$$\frac{\varepsilon_{nl}}{\varepsilon_{nl}-\beta^2}\,\partial_z E_x\,|_{z=0^+} - \frac{\varepsilon_s E_{x0}}{q} = -\sigma_{xx} E_{x0} \quad \text{or} \tag{40}$$

$$\varepsilon_{nl}\sqrt{qN^2\gamma}\,\tanh(\sqrt{\gamma N^2}\,(\bar{z})) - \varepsilon_s(\varepsilon_{nl}-\beta^2) = -\sigma_{xx} q \ \text{or} \tag{41}$$

$$(\frac{\varepsilon_{nl}}{\varepsilon_{nl}-\beta^2})((\varepsilon_{0x}+\sigma'_{xx})\left(\frac{\beta^2}{\varepsilon_{0x}}-1\right)(E_{x0})^2 + \alpha\left(\frac{\beta^2}{\varepsilon_{0x}}-1\right)\left(\frac{(E_{x0})^4}{2}\right) - \frac{\varepsilon_s E_{x0}}{q} + \sigma_{xx} E_{x0} = 0 \tag{42}$$

assuming $E_{x0}\,|_{z=0^-} = E_x\,|_{z=0^+} = E_{x0}$ and $\varepsilon_{nl} = \varepsilon_{0x} + \alpha E_{x0}{}^2$. In the case of the TM mode there is no

B_{jn}, j=1,2 field to be considered in this case. Using equations (20 and 21), we can calculate the power in the nonlinear medium using the equation for the Polynting vector

$$\langle \vec{S} \rangle = \frac{1}{2}\mathrm{Re}\left(\vec{E}\times\tilde{\vec{H}}\right) \tag{43}$$

and

$$P = \int_{-\infty}^{\infty}\langle S\rangle\cdot\hat{x} = \sqrt{\frac{\varepsilon_0}{\mu_0}}\,\frac{14\alpha}{3\gamma^{1/2}\left(\frac{\beta^2}{\varepsilon_{0x}}-1\right)^{3/2}}\,. \tag{44}$$

The nonlinear equations for the TM and TE modes have analytical solutions that have surface wave characteristics. There solutions are similar to homogenous NLSE. These functions also account for conductivity which is a complex quantity. The above nonlinear equations accounted for self-focusing and defocusing. A dispersion relation for nonlinear surface waves with conductivity was derived but one has to match nonlinear intensity enhancement with SP excitation with graphene conductivity which is was not part of this study.

3. OPTICAL BISTABILITY

Switching states in nonlinear optic devices are a subject of intense investigation recently in optics[6,15,21,24,32]. Optical devices are being studied and engineered to eventually replace electronic devices and carry out the same function but with better performance. This research has been further explored in nanotechnology or nanostructure devices at the subwavelength scale. Optical nonlinearity which is mostly studied at the microscale is applicable at the nanoscale. Electric Field enhanced dielectrics such as the Kerr effect can be useful in creating switching states in a dielectric/nonlinear Kerr medium, dielectric/metallic/nonlinear Kerr medium, and even a dielectric/thin graphene/nonlinear Kerr medium interface. These interfaces have optical bistable effects because of abrupt discontinious jumps in solutions of the $\vec{E}$ and $\vec{H}$ fields or reflection (transmission) coefficients. Here, the purpose is to show bistable states and hysteresis with the excitation of surface plasmons establish a basis for a optical switching.

First in nonlinear optics, nonlinear polarization displacement is

$$\vec{D} = \varepsilon\vec{E} \tag{45}$$

and

$$\vec{D} = \vec{E} + 4\pi\vec{P}. \tag{46}$$

For an isotropic medium ignoring the other tensor terms which are zeros, so polarization now is

$$\vec{P} = \mathrm{X}_1\vec{E} + \mathrm{X}_3\left|\vec{E}\right|^2\vec{E} \tag{47}$$

with X_1 and X_3 being susceptibilities of nonlinearities of polarization. The constants in equation (47) such that

$$\varepsilon = 1 + 4\pi\mathrm{X}_1 + 4\pi\mathrm{X}_3\left|E\right|^2 = \varepsilon^0 + \alpha\left|E\right|^2. \tag{48}$$

The constants ε, ε^0 and α are the nonlinear dielectric, dielectric, and nonlinear index of refraction of the Kerr medium.

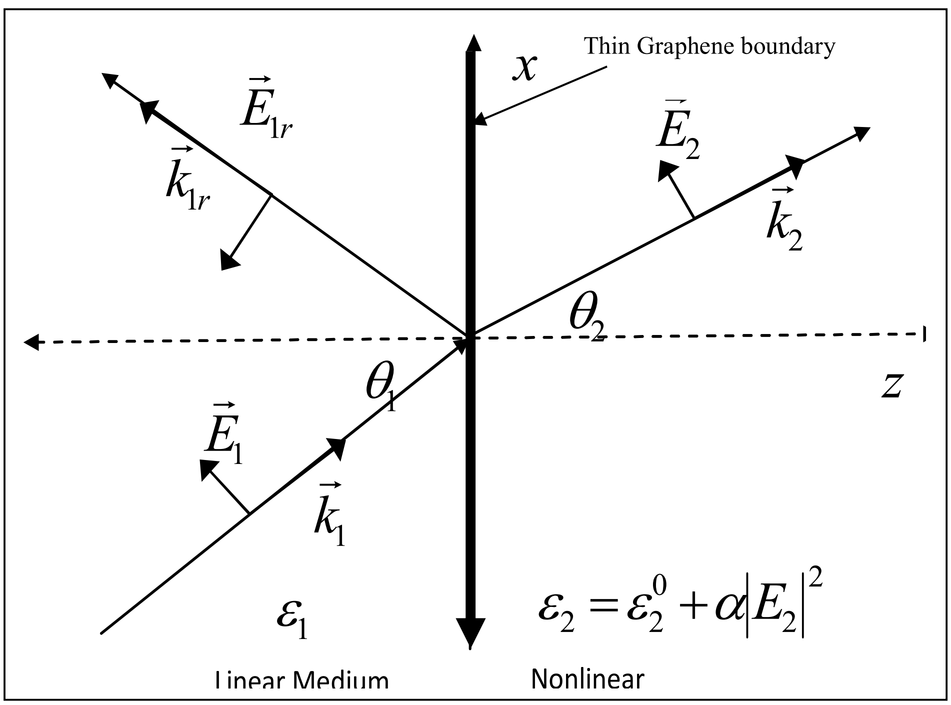


Figure 1. Transverse Magnetic (TM) incident, reflected, and transmitted Electric Fields.

Table 1.

TM Fields (with ω=frequency, c=speed of light)

Incident waves

$$E_{1x} = E_1 \cos(\theta_1) \exp i(k_{1x}x + k_{1z}z)$$
$$E_{1y} = 0$$
$$E_{1z} = -E_1 \sin(\theta_1) \exp i(k_{1x}x + k_{1z}z)$$
$$H_{1x} = 0$$
$$H_{1y} = \frac{c}{\omega} k_1 E_1 \exp i(k_{1x}x + k_{1z}z)$$
$$H_{1z} = 0$$

Reflected wave s

$$E_{1rx} = -E_{1r} \cos(\theta_1) \exp i(k_{1x}x - k_{1z}z)$$
$$E_{1ry} = 0$$
$$E_{1rz} = -E_{1r} \sin(\theta_1) \exp i(k_{1x}x - k_{1z}z)$$
$$H_{1x} = 0$$
$$H_{1ry} = \frac{c}{\omega} k_1 E_{1r} \exp i(k_{1x}x - k_{1z}z)$$
$$H_{1rz} = 0$$

Transmitted waves

$$E_{2x} = E_2(z) \cos(\theta_2) \exp i(k_{2x}x) \exp i(\int_0^z k_{2z} dz')$$
$$E_{1y} = 0$$
$$E_{2z} = -E_2(z) \sin(\theta_2) \exp i(k_{2x}x) \exp i(\int_0^z k_{2z} dz')$$
$$H_{1x} = 0$$
$$H_{2y} = \frac{c}{\omega}(k_2 E_2(z) - i\cos(\theta_2)\frac{dE_2}{dz}) \exp i(k_{2x}x) \exp i(\int_0^z k_{2z} dz')$$
$$H_{1z} = 0$$

Considering figure (1), the simple dielectric/nonlinear interface using Maxwell's Equations in (cgs units), the fields are represented in Table 1. The fields are incident, reflected, and transmitted waves with directional k_1 k_2 wave vectors. All fields are assumed to be exp(-iωt) time dependent. The magnetic fields H_{1y} and H_{2y} calculated in medium 2 is based on a component of Faraday's Law in the frequency domain

$$H_{1y} = -\frac{ic}{\omega}\left(\frac{\partial E_{1x}}{\partial z} - \frac{\partial E_{1z}}{\partial x}\right)$$

$$H_{2y} = -\frac{ic}{\omega}\left(\frac{\partial E_{2x}}{\partial z} - \frac{\partial E_{2z}}{\partial x}\right). \tag{49}$$

The fields in the nonlinear medium are in the solution of the form

$$\vec{E}_2 = \vec{E}_2(z)\exp i(k_{2x}x)\exp i\left(\int_{z_1}^{z_2} k_{2z}dz'\right) \tag{50}$$

with again, Faraday's Law in vector form

$$\frac{i\omega}{c}\vec{H}_2 = \vec{\nabla}\times\vec{E}_2. \tag{51}$$

The nonlinear Helmholtz wave equation is

$$-\nabla^2\vec{E}_2 + \vec{\nabla}\left(\vec{\nabla}\cdot\vec{E}_2\right) - \frac{\omega^2}{c^2}\left(\varepsilon_2^0 + \alpha\left|E_2\right|^2\right)\vec{E}_2 = 0 \tag{52}$$

Equation (52) solved numerically but here the field will be approximated by equation (50). The equation can be approximated to model solitary propagation [27] and nonlinear surface waves (in previous section 2.1). The components of the wave vectors given by

$$k_1 = \frac{\omega\sqrt{\varepsilon_1}}{c} \qquad k_2 = \frac{\omega\sqrt{\varepsilon_2}}{c}$$

$$k_{2x}^2 + k_{2z}^2 = k_2^2 = \frac{\omega^2}{c^2}\varepsilon_2 \tag{53}$$

and

$$k_{1x} = \frac{\omega}{c}\sqrt{\varepsilon_1}\sin\theta_1 \qquad k_{1z} = k_1\cos\theta_1 \tag{54}$$

$$k_{2x} = \frac{\omega\sqrt{\varepsilon_2}}{c}\sin\theta_2 \qquad k_{2z} = k_2\cos\theta_2$$

According to figure (1) and table (1), we apply boundary conditions of the electric and fields at z=0 resulting the relations

$$E_1\cos\theta_1\exp(ik_{1x}x) - E_{1r}\cos\theta_1\exp(ik_{1x}x) = E_2(0)\cos\theta_2\exp(ik_{2x}x) \tag{55}$$

$$k_1E_1\exp(ik_{1x}x) + k_1E_{1r}\exp(ik_{1x}x) = \left[k_2E_2(0) - i\cos\theta_2\frac{dE_2}{dz}\right]\exp(ik_{2x}x) \tag{56}$$

In equation (56), the second term on the right hand side is approximated to zero due to the slow vary electric field amplitude E_2 as function of z. For a boundary condition,

$$k_{2x} = k_{1x} = \frac{\omega}{c}\sqrt{\varepsilon_1}\sin\theta_1 \tag{57}$$

and

$$k_{2z} = \frac{\omega}{c}\sqrt{\varepsilon_2 - \varepsilon_1 \sin^2\theta_1}\,. \tag{58}$$

Keeping in memory that the transmitted angle θ_2 in the nonlinear medium can be complex so the k_2 and k_{2z} in equation have to be calculated. The critical angle θ_c for TIR (total internal reflection) is

$$\varepsilon_2 = \varepsilon_1 \sin^2\theta_c \tag{59}$$

$$\varepsilon_2 < \varepsilon_1 \sin^2\theta_1 \qquad \text{for TIR} \tag{60}$$

or

$$k_{2z} = i\frac{\omega}{c}\sqrt{\varepsilon_1 \sin^2\theta_1 - \varepsilon_2}\,. \tag{61}$$

Substituting equation (55) into equation (56) with using equations (57 and 58), the Fresnel relations are

$$E_1 = E_2(0)\frac{\sqrt{\varepsilon_2}\cos(\theta_1) + \sqrt{\varepsilon_1}\sqrt{1 - \frac{\varepsilon_1}{\varepsilon_2}\sin^2(\theta_1)}}{2\sqrt{\varepsilon_1}\cos(\theta_1)} \tag{62}$$

$$E_{1r} = E_2(0)\frac{\sqrt{\varepsilon_2}\cos(\theta_1) - \sqrt{\varepsilon_1}\sqrt{1 - \frac{\varepsilon_1}{\varepsilon_2}\sin^2(\theta_1)}}{2\sqrt{\varepsilon_1}\cos(\theta_1)} \tag{63}$$

with $\varepsilon_2 = \varepsilon_2^0 + \alpha\left|E_2\right|^2$. Equations (62-63) are in terms of the incident angle, the dielectric constant, and the nonlinear dielectric medium. Referring back to equations (36-37), equations (55-56) and figure (1), a very thin layer of graphene can be approximated to conductivity [16] such that

$$\vec{H}_{1t} - \vec{H}_{2t} = \vec{J}_s \times \hat{n} = \overline{\overline{\sigma}}'\vec{E} \tag{64a}$$

and the $\vec{J}_s \times \hat{n}$ is oriented

$$\sigma_{xx}E_x\hat{x}\times\hat{n} = -\sigma_{xx}E_x\hat{y} \tag{64b}$$

for the TM mode. Boundary condition equation with conductivity ($\sigma \equiv \sigma_{xx}$ from the conductivity tensor) are

$$E_1 \cos\theta_1 - E_{1r} \cos\theta_1 = E_2(0)\cos\theta_2 \tag{65}$$

$$k_1 E_1 + k_1 E_{1r} = k_2 E_2(0) - \sigma E_1 \cos\theta_1 . \tag{66}$$

This leads to the following Fresnel equations

$$E_1 = E_2(0)\frac{\sqrt{\varepsilon_2}\cos(\theta_1) + \sqrt{\varepsilon_1}\sqrt{1 - \frac{\varepsilon_1}{\varepsilon_2}\sin^2(\theta_1)}}{(2+\sigma\cos(\theta_1))\sqrt{\varepsilon_1}\cos(\theta_1)} \tag{67}$$

$$E_{1r} = E_2(0)\frac{\sqrt{\varepsilon_2}\cos(\theta_1) - \sqrt{\varepsilon_1}\sqrt{1 - \frac{\varepsilon_1}{\varepsilon_2}\sin^2(\theta_1)} - \sigma\sqrt{\varepsilon_2}\cos^2(\theta_1)}{(2+\sigma\cos(\theta_1))\sqrt{\varepsilon_1}\cos(\theta_1)} \tag{68}$$

With $\sigma=0$, equations (67-68) return to equations (62-63). The conductivity for graphene media has to be carefully handled due its dependency on frequency, temperature, chemical potential, and electron energy. Graphene conductivity physics is a separate area popular research [16,26]. The integral for graphene conductivity [16] is

$$\sigma(\omega, \mu_c, \gamma, T) = ie^2 \frac{(\omega + i2\gamma)}{\pi\hbar^2}\left\{\frac{1}{(\omega+i2\gamma)^2}\int_0^\infty \mathrm{N}\left[\frac{\partial f_d(\mathrm{N})}{\partial \mathrm{N}} - \frac{\partial f_d(-\mathrm{N})}{\partial \mathrm{N}}\right] d\mathrm{N} .\right.$$

$$\left. - \int_0^\infty \frac{f_d(-\mathrm{N}) - f_d(\mathrm{N})}{(\omega+i2\gamma)^2 - 4\left(\frac{\mathrm{N}}{\hbar}\right)^2} d\mathrm{N}\right\}, \quad \text{where} \tag{69}$$

$$f_d(\mathrm{N}) = \left[\exp\left(\frac{\mathrm{N}-\mu_c}{k_B T}\right)_c + 1\right]^{-1} .$$

The constants ***e***, γ, μ_c, N, $\hbar$, and ***T*** are electron charge, decay constant, chemical potential, electron energy, and temperature. The conductivity is characterized by interband and intraband transitions in the in the conduction and valance bands of graphene [28]. Equation (69) are results set in the complex domain so for certain frequencies the imaginary part of the conductivity becomes negative which means TE (Transverse Electric) surface waves can propagate along a waveguide graphene layer. At other frequencies only TM surface waves propagate along the graphene layer. However, this does not necessary account for a behavior of a graphene layer with a nonlinear Kerr medium. Metallic/Dielectric interfaces for waveguides do not support SPs TE modes [3,19]. Since the graphene conductivity is a complex number, the real and imaginary parts can be treated with care as dielectric equivalent similar to

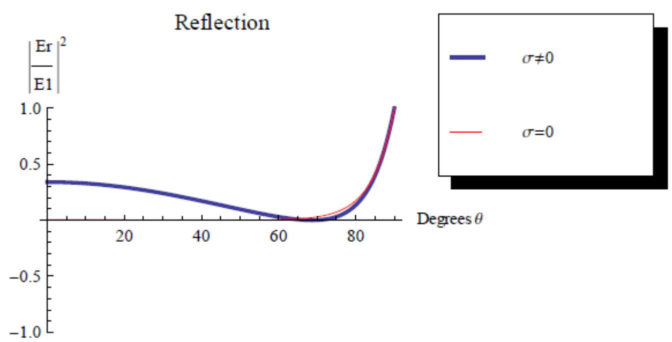
$$\sigma_{graphene} \equiv \sigma_r + i\sigma_i \tag{70}$$

$$\sigma_{graphene} \equiv \left(-\frac{\sigma_r}{\omega\Delta} + i\frac{\sigma_i}{\omega\Delta} \right)|_{\Delta \to 0} \equiv \varepsilon_{r.eq} + i\varepsilon_{i,eq} \tag{71}$$

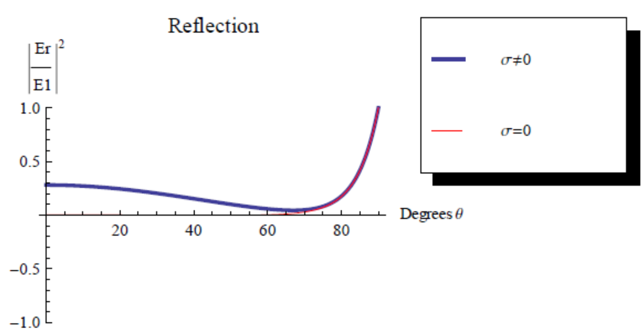
This dielectric can approximate to 1 atomic layer in a waveguide which is part of the boundary condition instead of another waveguide layer [29]. In figure(2), the is a comparison Reflection coefficients from Fresnel's Equations (62-63 and 67-68) of the single interface dielectric/dielectric medium interface with and without conductivity. The reflection coefficient is

$$R \equiv \left| \frac{E_{1r}}{E_1} \right|^2. \tag{72}$$

In figure (2), graphs (c) and (d) show total reflection greater 60 degrees with and without conductivity interface (equations 62-63, 67-68). The figures also include $\varepsilon_1 > \varepsilon_2$ and $\varepsilon_1 < \varepsilon_2$. The reflection equation (72) is not a good measure to handle nonlinear Kerr medium since the intensity is dependent on electric fields. This is similar problem with dispersion relations with nonlinear media [1, 6, and 15].



(a)



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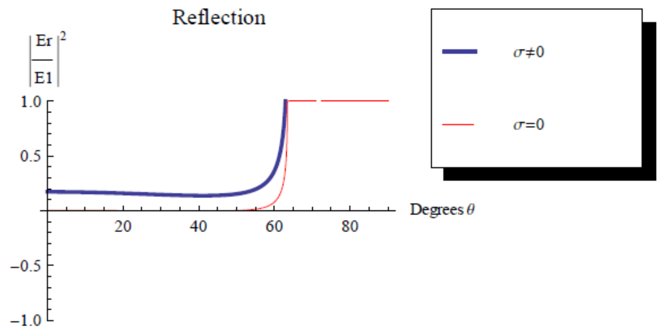
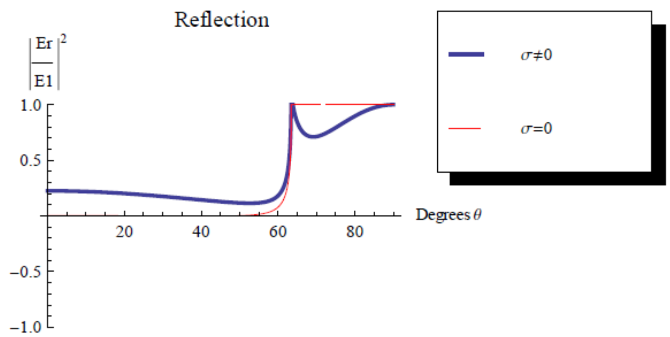


(c)



(d)

Figure (2). Reflection coefficients of Electric Fields with linear dielectrics with and without conductivity (a) σ=-1and (b) σ=1i,with ε_1=5, ε_2=4 (c)) σ=-1 and (d)) σ=1i, with ε_1=4, ε_2=5.

A better approach would to define the dimensionless intensity [4]

$$U_1 \equiv \frac{\alpha}{\varepsilon_2^0}\left|E_1\right|^2 \tag{73}$$

$$U_{1r} \equiv \frac{\alpha}{\varepsilon_2^0}\left|E_{1r}\right|^2 \tag{74}$$

$$U_2 \equiv \frac{\alpha}{\varepsilon_2^0}\left|E_2\right|^2 \tag{75}$$

which are incident, reflected, and transmitted nonlinear measures knowing

$$\varepsilon_2 = \varepsilon_2^0 + \alpha\left|E_2\right|^2 \quad \text{and} \tag{76}$$

$$\varepsilon \equiv \frac{\varepsilon_2^0}{\varepsilon_1} = \left(\frac{n_2^0}{n_1}\right)^2 = \sin^2(\theta_c^0). \tag{77}$$

Also the quantities can be converted back in terms of the Poynting vector

$$\left|\vec{S}_1\right| = \frac{c}{16\pi}\frac{\sqrt{\varepsilon_2^0\varepsilon_1}}{n_{nl}}U_1 \text{ with } n_{nl} = \frac{\alpha}{2\sqrt{\varepsilon_2^0}}. \tag{78a}$$

For a Kerr –like medium equations (73-76) can be defined as

$$U_1 \equiv \frac{\alpha}{\varepsilon_2^0}\left|E_1\right|^{2\nu} \tag{78b}$$

$$U_{1r} \equiv \frac{\alpha}{\varepsilon_2^0}\left|E_{1r}\right|^{2\nu} \tag{78c}$$

$$U_2 \equiv \frac{\alpha}{\varepsilon_2^0}\left|E_2\right|^{2\nu} \tag{78d}$$

which are incident, reflected, and transmitted nonlinear measures knowing

$$\varepsilon_2 = \varepsilon_2^0 + \alpha\left|E_2\right|^{2\nu} \quad \text{and} \quad 0 < \nu \le 1. \tag{79e}$$

The angle θ_c^0 is the critical angle when nonlinear constant to the intensity is not present (α=0). The ensuing equations based on equations (73-77)

$$U_1 = \frac{1}{4}U_2\left|\sqrt{\varepsilon(1+U_2)} + \sec(\theta_1)\sqrt{1-\sin^2(\theta_1)/\varepsilon(1+U_2)}\right|^2 \tag{79}$$

$$U_{1r} = \frac{1}{4}U_2\left|\sqrt{\varepsilon(1+U_2)} - \sec(\theta_1)\sqrt{1-\sin^2(\theta_1)/\varepsilon(1+U_2)}\right|^2 \tag{80}$$

for transmission,

$$U_1 = \frac{1}{4}U_2\left|\sqrt{\varepsilon(1+U_2)} + i\sec(\theta_1)\sqrt{\frac{\sin^2(\theta_1)}{\varepsilon(1+U_2)} - 1}\right|^2 \tag{81}$$

$$U_{1r} = \frac{1}{4}U_2\left|\sqrt{\varepsilon(1+U_2)} - i\sec(\theta_1)\sqrt{\frac{\sin^2(\theta_1)}{\varepsilon(1+U_2)} - 1}\right|^2 \tag{82}$$

for TIR and with σ≠0

$$U_{1r} = U_2 \left| \frac{1}{(2+\sigma\cos(\theta_1))} \left(\sqrt{\varepsilon(1+U_2)} - \sec(\theta_1)\sqrt{1-\frac{\sin^2(\theta_1)}{\varepsilon(1+U_2)}} - \sigma\cos(\theta_1)\sqrt{\varepsilon(1+U_2)} \right) \right|^2 \quad (83)$$

$$U_1 = U_2 \left| \frac{1}{(2+\sigma\cos(\theta_1))} \left(\sqrt{\varepsilon(1+U_2)} + \sec(\theta_1)\sqrt{1-\frac{\sin^2(\theta_1)}{\varepsilon(1+U_2)}} \right) \right|^2 \quad (84)$$

$$U_{1r} = U_2 \left| \frac{1}{(2+\sigma\cos(\theta_1))} \left(\sqrt{\varepsilon(1+U_2)} - i\sec(\theta_1)\sqrt{\frac{\sin^2(\theta_1)}{\varepsilon(1+U_2)}-1} - \sigma\cos(\theta_1)\sqrt{\varepsilon(1+U_2)} \right) \right|^2 \quad (85)$$

$$U_1 = U_2 \left| \frac{1}{(2+\sigma\cos(\theta_1))} \left(\sqrt{\varepsilon(1+U_2)} + i\sec(\theta_1)\sqrt{\frac{\sin^2(\theta_1)}{\varepsilon(1+U_2)}-1} \right) \right|^2 . \quad (86)$$

Now nonlinear reflection can be calculated

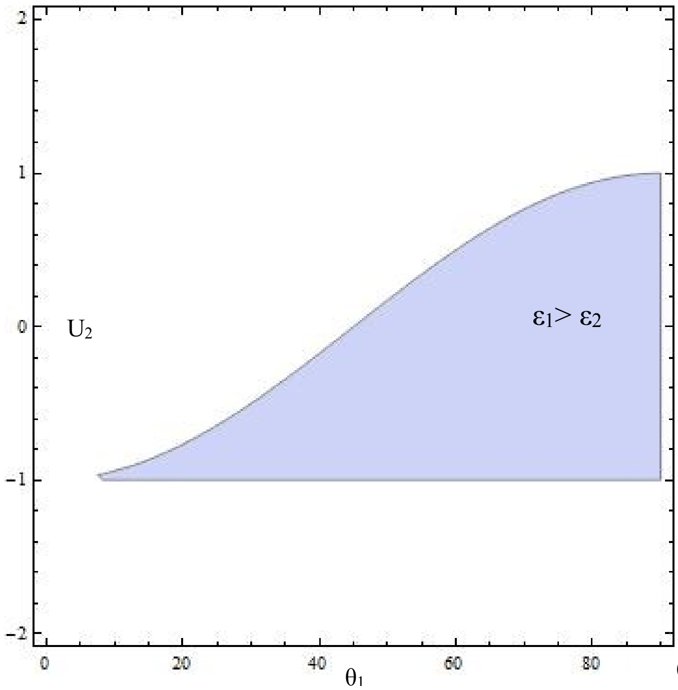
$$R = \frac{U_{1r}}{U_1}. \quad (87)$$

In equations (79-82), U_{1r} and U_1 as functions of U_2 can be determined with the incident angle θ_1 and the dielectric ε fixed. Equations (79-80 and 84-85) are real (transmission) and (82-83 and 85-86) are imaginary (TIR). In equations (81-82) when R=1 ($U_{1r} = U_1$)

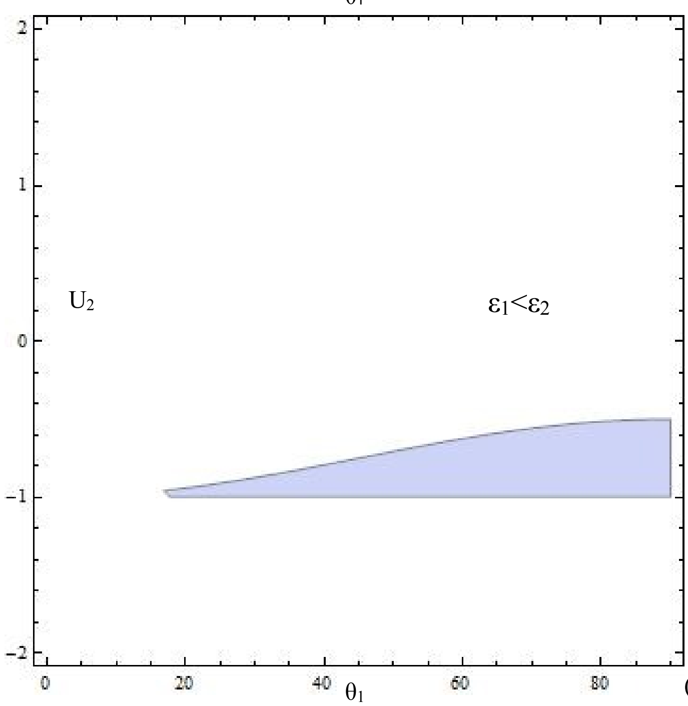
$$R = \frac{\left(\varepsilon(1+U_2) - \sec^2(\theta_1)\left(\frac{\sin^2(\theta_1)}{\varepsilon(1+U_2)} - 1 \right) \right)}{\left(\varepsilon(1+U_2) + \sec^2(\theta_1)\left(\frac{\sin^2(\theta_1)}{\varepsilon(1+U_2)} - 1 \right) \right)} \quad (88)$$

In order for TIR mode to exist in the nonlinear medium,

$$U_2 \le \frac{\sin^2(\theta_1)}{\varepsilon} - 1 \quad (89)$$



(a)



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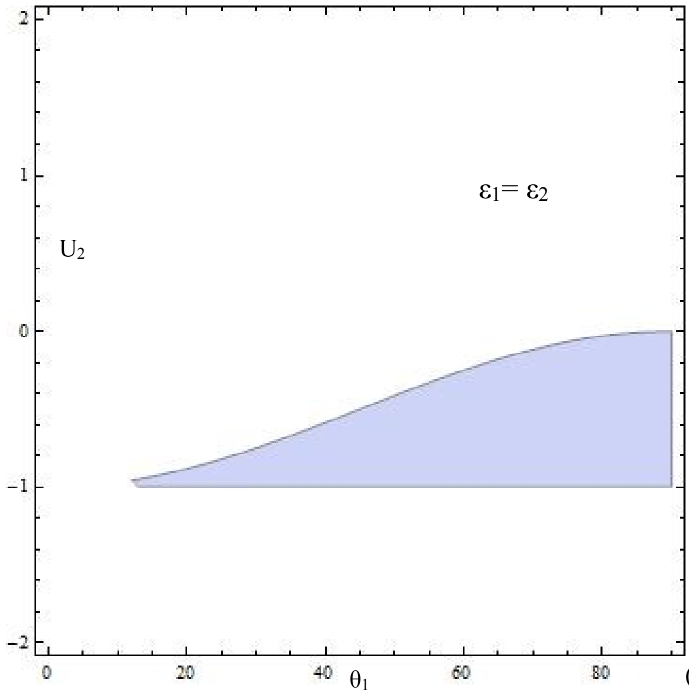

Figure 3. $U_2 \le \frac{\sin^2(\theta_1)}{\varepsilon} - 1$ (a),(b), and (c) for $\varepsilon_1=2.0, \varepsilon_2=1.0$

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If U_2 is equated in (equation 89) and solving for nonlinear critical intensity U_1^C

$$U_1^C = \frac{1}{4}\left(\frac{\sin^2(\theta_1)}{\varepsilon} - 1\right)\sin^2(\theta_1). \tag{90}$$

The TIR mode condition is more stringent with conductivity counted in equations (83-86). Assume conductivity to be imaginary σ=iσ', so

$$U_2 \ge \left[\varepsilon\left(\frac{1}{\tan^2(\theta_1)} + 1\right)\right]^{-1} - 1 \tag{91}$$

$$\sigma' = -2\sqrt{\frac{\tan^2(\theta_1)}{(\varepsilon(1+U_2))^2} - \frac{\sec^2(\theta_1)}{(\varepsilon(1+U_2))}} \tag{92}$$

In this case both expressions above have to be substituted back into equation (85-86) to get the nonlinear critical intensity U_1^C with conductivity. For the Transverse Electric (TE) case, expressions for the intensities with conductivity are

$$U_1 = U_2 \left| \frac{\cos(\theta_1)\sqrt{\frac{1}{\varepsilon(1+U_2)}} + \sqrt{\varepsilon_1}\sec(\theta_1)\sqrt{1-\frac{\sin^2(\theta_1)}{\varepsilon(1+U_2)}}}{2\sqrt{\varepsilon_1}\cos(\theta_1)-\sigma_{yy}} \right|^2 \tag{93}$$

$$U_{1r} = U_2 \left| \frac{\cos(\theta_1)\sqrt{\frac{1}{\varepsilon(1+U_2)}} + \sqrt{\varepsilon_1}\sec(\theta_1)\sqrt{1-\frac{\sin^2(\theta_1)}{\varepsilon(1+U_2)}}}{2\sqrt{\varepsilon_1}\cos(\theta_1)-\sigma_{yy}} \right. \cdot$$

$$\left. \frac{\sqrt{\varepsilon_1}\cos(\theta_1)\sqrt{\frac{1}{\varepsilon(1+U_2)}} - \sqrt{\varepsilon_1}\sqrt{1-\frac{\sin^2(\theta_1)}{\varepsilon(1+U_2)}} - \sigma_{yy}\sqrt{\frac{1}{\varepsilon(1+U_2)}}}{\sqrt{\varepsilon_1}\sqrt{1-\frac{\sin^2(\theta_1)}{\varepsilon(1+U_2)}} - \sqrt{\varepsilon_1}\cos(\theta_1)\sqrt{\frac{1}{\varepsilon(1+U_2)}}} \right|^2 \tag{94}$$

Since the waves are TE or $\left(E_z = E_x = H_y = 0\right)$, conductivity is oriented

$$\vec{J}_s \times \hat{n} = \sigma_{yy} E_y \hat{y} \times \hat{n} = \sigma_{yy} E_y \hat{x} \tag{95}$$

Again, in metallic/dielectric interface surface plasmons do not exist in TE mode, but they exist for this mode on a graphene dielectric interface or a multilayer graphene/dielectric waveguide [16,26]. In order for bistability to occur for one value U_1 there should be multiple values of U_2 leading to different values of U_{1r}. This expression for intensity can increase or decrease with no change in slope so a systematic approach to finding bistability is looking extremum points. In the case of conductivity being zero equations (81-82) bistability exist with the nonlinear coefficient being negative (or $\alpha<0$). To check for slope change or switching points

$$\frac{dU_1}{dU_2}\Big|_{U_1=U_1^s} = 0 \tag{96}$$

and the incident intensity approximated

$$U_1 \approx \frac{1}{4}U_2\left[\varepsilon(1+U_2) + \sec^2(\theta_1)(1-\frac{\sin^2(\theta_1)}{\varepsilon}(1-U_2) + 2\sec(\theta_1)\sqrt{\varepsilon(1+U_2)-\sin^2(\theta_1)}\right] \tag{97}$$

with the zero slope value and switching value (ε=1)

$$U_2^s \approx \left[-\left(1+\frac{\sqrt{\varepsilon-\sin^2(\theta_1)}}{\varepsilon\cos(\theta_1)}\right)^2 \left(2\left(1+\frac{\tan^2(\theta_1)}{\varepsilon}\right) + \frac{3\sec(\theta_1)}{\sqrt{\varepsilon-\sin^2(\theta_1)}}\right)^{-1} \right] \tag{98}$$

$$U_1^s \approx -42\cos^2(\theta_1). \tag{99}$$

If conductivity (with ε=1) is in the boundary conditions for TM mode the switching values are

$$U_2^{s1,s2} = \left[\left(-2-2^{\frac{3}{2}}Tan^{-3}(\theta_1)\right) \pm \left[\left(\cos(\theta_1)-\sec^2(\theta_1)-2-2^{\frac{3}{2}}Tan^3(\theta_1)-Tan^2(\theta_1)\right)^2\right.\right.$$

$$\left.\left. -\left(2\cos^2(\theta_1)+\frac{3}{2}2^{\frac{3}{2}}Tan^{-3}(\theta_1)\right)\left(3+\sec^2(\theta_1)+Tan^2(\theta_1)\right)\right]^{\frac{1}{2}}\right]\left(-3-\cos^2(\theta_1)-3\cdot 2^{\frac{3}{2}}Tan^{-3}(\theta_1)\right)^{-1} \tag{100}$$

In order to deal the nonlinear dielectric that is intensity dependent the nonlinear coefficient and the dielectric constant of the nonlinear dielectric has to be dimensionless measure to properly handle reflection and transmission quantities. By calculating the dielectric constant integral for graphene based on frequency, chemical potential, temperature, electron energy the reflection and transmission for intensities can be calculated with nonlinear waveguide. Lastly, one has to be able to take advantage of conductivity and required quantity of intensity for the Kerr effect to study the full features of this waveguide.

4. MULTILAYER GRAPHENE AND ONE LAYER KERR MEDIUM

In the previous section the single layered dielectric/nonlinear Kerr dielectric with a thin graphene layer approximated to a boundary condition was presented to show optical bistability but here a multilayered configuration(figure 4 below), dielectric/dielectric/nonlinear Kerr dielectric with multiple thin graphene layers is considered. Similar incident, reflected, and transmitted waves functions for TM and TE modes are assumed. The approximated wave in the Kerr medium 3 is

$$H_{3y} = \frac{c}{\omega}(k_3 E_3(z) - i\cos(\theta_3)\frac{dE_3(z)}{dz})\exp i(k_{3x}x)\exp i(\int_d^z k_{3z}dz') \tag{101}$$

with the constant d being the distance between medium1 and 3. Again, for this case the derivative term in equation (101) is approximated to zero since it is slowly varying amplitude. The angles in medium 2 and 3 can be complex so the wave vectors relations are

$$k_1 = \frac{\omega\sqrt{\varepsilon_1}}{c} \; k_2 = \frac{\omega\sqrt{\varepsilon_2}}{c} \; k_2 = \frac{\omega\sqrt{\varepsilon_3^0 + \alpha|E_3(z)|^2}}{c} \tag{102}$$

$$k_{2x}^2 + k_{2z}^2 = k_2^2 = \frac{\omega^2}{c^2}\varepsilon_2$$

$$k_{1x} = k_{2x} = k_{3x} \tag{103}$$

and

$$k_{1x} = \frac{\omega}{c}\sqrt{\varepsilon_1}\sin\theta_1 \qquad k_{1z} = k_1\cos\theta_1$$

$$k_{2x} = \frac{\omega\sqrt{\varepsilon_2}}{c}\sin\theta_2 \qquad k_{2z} = k_2\cos\theta_2$$

$$k_{3x} = \frac{\omega}{c}\sqrt{\varepsilon_1}\sin\theta_1 \qquad k_{3z} = k_1\cos\theta_1 \tag{104}$$

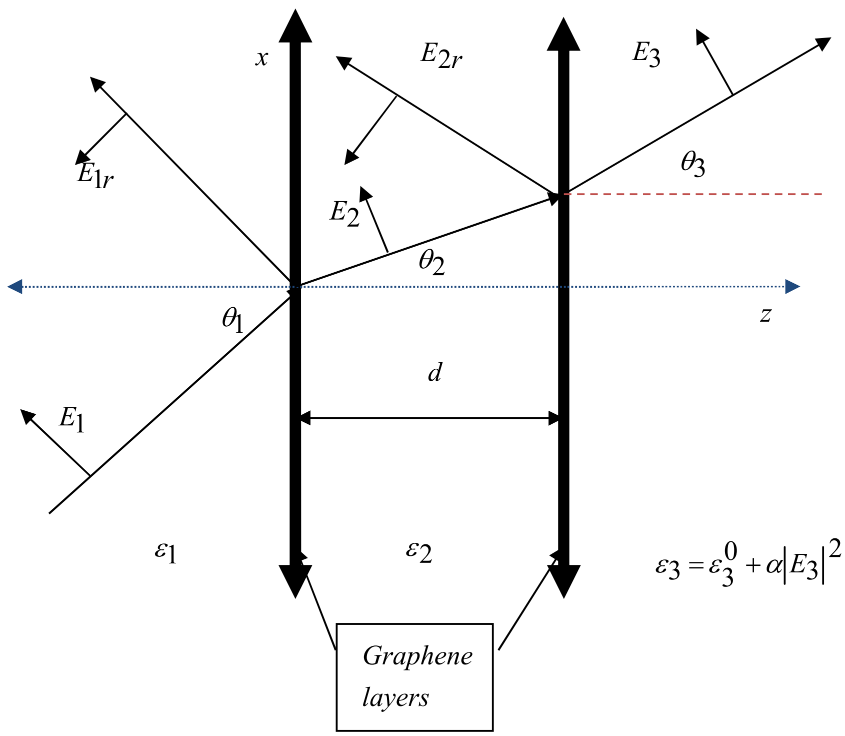


Figure 4. The multilayered waveguide with two thin graphene layers.

the wave vectors relations are

$$k_1 = \frac{\omega\sqrt{\varepsilon_1}}{c}\ k_2 = \frac{\omega\sqrt{\varepsilon_2}}{c}\ k_2 = \frac{\omega\sqrt{\varepsilon_3^0 + \alpha|E_3(z)|^2}}{c} \tag{105}$$

$$k_{2x}^2 + k_{2z}^2 = k_2^2 = \frac{\omega^2}{c^2}\varepsilon_2$$

$$k_{1x} = k_{2x} = k_{3x} \tag{106}$$

and

$$k_{1x} = \frac{\omega}{c}\sqrt{\varepsilon_1}\sin\theta_1 \qquad k_{1z} = k_1\cos\theta_1$$

$$k_{2x} = \frac{\omega\sqrt{\varepsilon_2}}{c}\sin\theta_2 \qquad k_{2z} = k_2\cos\theta_2$$

$$k_{3x} = \frac{\omega}{c}\sqrt{\varepsilon_1}\sin\theta_1 \qquad k_{3z} = k_1\cos\theta_1 \tag{107}$$

Apply pertinent boundary conditions and z=0and z=d with continuity of electric and magnetic fields for TM modes with and without conductivity (σ) in matrix form

$$\begin{bmatrix} 1 & \dfrac{\cos(\theta_2)}{\cos(\theta_1)} & -\dfrac{\cos(\theta_2)}{\cos(\theta_1)} & 0 \\ -1 & \dfrac{k_2}{k_1} & \dfrac{k_2}{k_1} & 0 \\ 0 & \exp(ik_{2z}d) & -\exp(-ik_{2z}d) & -\dfrac{\cos(\theta_3)}{\cos(\theta_2)} \\ 0 & \exp(ik_{2z}d) & \exp(-ik_{2z}d) & \dfrac{-k_3}{k_2} \end{bmatrix} \begin{bmatrix} E_{1r} \\ E_2 \\ E_{2r} \\ E_3(z) \end{bmatrix} = \begin{bmatrix} E_1 \\ E_1 \\ 0 \\ 0 \end{bmatrix} \tag{108}$$

$$\begin{bmatrix} 1 & \dfrac{\cos(\theta_2)}{\cos(\theta_1)} & -\dfrac{\cos(\theta_2)}{\cos(\theta_1)} & 0 \\ -\dfrac{k_1}{k_1+\sigma} & \dfrac{k_2}{k_1+\sigma} & \dfrac{k_2}{k_1+\sigma} & 0 \\ 0 & \exp(ik_{2z}d) & -\exp(-ik_{2z}d) & -\dfrac{\cos(\theta_3)}{\cos(\theta_2)} \\ 0 & \exp(ik_{2z}d) & \dfrac{k_2}{k_2+\sigma}\exp(-ik_{2z}d) & \dfrac{-k_3(d)}{k_2+\sigma} \end{bmatrix} \begin{bmatrix} E_{1r} \\ E_2 \\ E_{2r} \\ E_3(z) \end{bmatrix} = \begin{bmatrix} E_1 \\ E_1 \\ 0 \\ 0 \end{bmatrix} \tag{109}$$

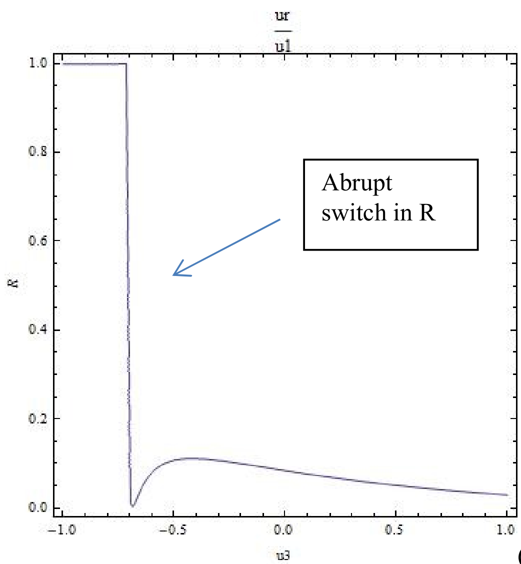
The plasmon angle [2-5] is given by

$$k_x^2 = \frac{\varepsilon_2\varepsilon_3}{\varepsilon_2+\varepsilon_3} = \varepsilon_1\sin^2\theta_p \tag{110}$$

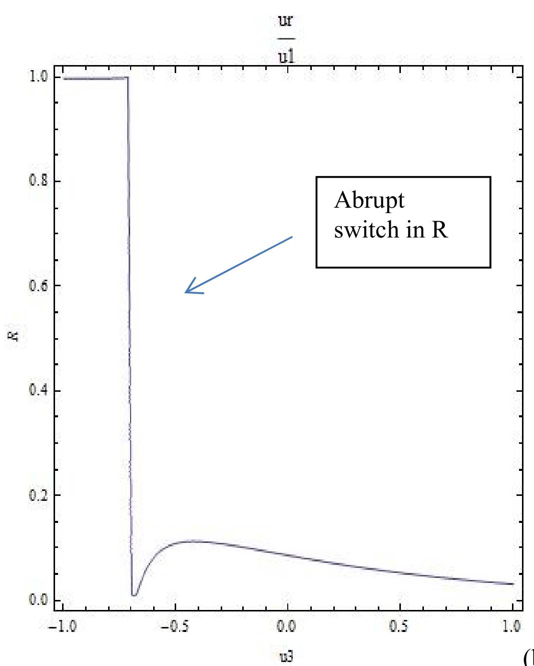
If we solve matrix equation (108) by reducing the matrix using Cramer's rule for calculating the determinants and solving for

$$R = \frac{U_{1r}(g(U_3)}{U_1(f(U_3))}. \tag{111}$$

Without using known matrix methods the algebra can be quite difficult. The figure(5) below shows an example of switching state from R=1 to close R ≈0 with a complex dielectric silver (1.06×10^{-6}m) in medium 2 and nonlinear medium 3 with measures of dimensionless intensities. This approach can also be taken for TE surface waves setting boundary condition equations with correct conductivity constant and orientation.



(a)



(b)

Figure (5). Reflection coefficients intensities with σ=0 with linear dielectrics with and without conductivity (a)θ_1 =53.76, ε_1=3.6, ε_2=-57.8+i.6, and ε^0_3=2.25 (b) θ_1 =53.90 ε_1=3.6, ε_2=-57.8+i.6, and ε^0_3=2.25.

5. CONCLUSION

Nonlinear surface wave propagation was presented in other order show that Kerr or Kerr-like material can support SP propagation especially with thin graphene layer between a dielectric and nonlinear dielectric. Using Maxwell's Equations and pertinent boundary conditions analytical solutions for TE and TM modes for self-focusing and self-defocusing, dispersion relation and expression for energy flux was derived. Next, reflection and transmission coefficients for a dielectric/nonlinear Kerr media presented to show that these quantities had to be calculated using dimensionless intensities that included the nonlinear coefficient and constant dielectric for that nonlinear medium. For the TM and TE mode reflection and transmission coefficients were presented that included a thin graphene layer. Briefly, the integral for dielectric of graphene presented. The incident frequency (among other quantities) impacts the conductivity of the graphene but the reaction of nonlinear Kerr material must be also considered. The approach to finding switching states for bistability would be to find the change in slope of the intensity equations that are used to calculate reflection and transmission coefficients. Lastly, an example for a multilayered configuration with multiple graphene thin layers was presented. The matrix equations were derived with and without conductivity in the TM mode. Reflection was calculated for multilayered configuration with silver in medium 2 (figure 5) to demonstrate switching in optical bistability.

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