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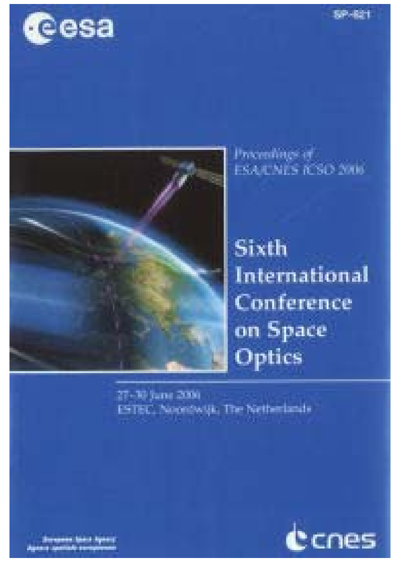


# *Laser frequency stabilisation for space applications*

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# LASER FREQUENCY STABILISATION FOR SPACE APPLICATIONS

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**ABSTRACT**
Laser frequency stabilisation is a necessity in all experiments demanding a metrological precision in the measurement of time, frequency, distance and displacement. Depending on the required performances of the laser source in term of precision, stability and timescale of said parameters a molecular or mechanical reference can be better suited to ensure the performance of the laser source. For both types of references, a brief overview of several locking techniques shall be presented. Special attention will be given to the implementation of both references and locking techniques, in a space environment. The technique of Tilt Locking (Gray, [4]) on a Fabry-Perot (FP) cavity shall be discussed in detail, given its a priori excellent adaptability to a space environment (no active components and simplified optical design).

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## 1. INTRODUCTION

Depending on the application and the needs, frequency stabilisation can require different references and locking techniques.
To start with, we can distinguish between long term and short term stability, the first one measured over times duration larger than 1s; the latter measured over times smaller than 1s. Another important parameter to be taken into account is the precision as it is requested in formation flight experiments.
The short term frequency stability of a laser is usually estimated thanks to the Signal to Noise Ratio and is best obtained with the use of a mechanical reference whose linewidth can be as narrow as $10^2$ Hz. For short term stabilisation we ask only that the slope of the error signal be steep, which implies that the linewidth must be narrow. This is verified for the mechanical cavities were the linewidth of the reference (<< 1 kHz) is narrower than what we can obtain with molecules (1-100 kHz). For stabilization on cavities we can increase laser powers to reduce the shot noise. The power usable with molecular references, instead, is limited by the saturation broadening, which for high values of the laser power increases the linewidth of the transition. For equivalent values of the photon noise we shall have much larger linewidths for molecules than obtained with the mechanical references. Resonators are to be preferred for short term stabilisation.

For long term stabilisation, instead, we ask that the frequency of the laser source do not vary significantly over time intervals of over 1000s. These time variations shall mostly depend on the reference frequency drifts but not on its linewidth. It is, therefore, obvious that for long term frequency stabilisation molecular and atomic transitions are favoured. Molecular linewidth can change significantly ($10^4$Hz/K) with the pressure (this parameter is in its turn temperature dependent) but the central position of the reference will be independent (typically below $10^2$Hz/K) at first order from the thermodynamic parameters of the gas. On the length of a Fabry-Perot, instead, changes of the order of $\Delta L / L = \alpha \Delta T$ (which shall give, for the frequency, equivalent variations of 1MHz/K) can be expected to intervene, even for an Ultra Low Expansion glass with $\alpha=1.9\cdot10^{-9}$ $(T-T0)<10^{-8}K^{-1}$.
Reproducibility and exactitude require a molecular or atomic reference, since their long term variations are smaller and their transitions proper frequencies are known. Even if the FP proper frequency were measured before the start of a space mission it still would be possible for a laser frequency to be more than one FSR (typically a GHz) away from its chosen frequency. Where an absolute knowledge of the laser's frequency is required a molecular reference must perforce be used.

## 2. FREQUENCY LOCKING ON A MECHANICAL REFERENCE : POUND-DREVER-HALL

Frequency locking on a Fabry-Perot cavity is most often accomplished through use of the Pound-Drever-Hall [1] technique.
Invented by Pound in the micro-wave range, it was applied by Drever and Hall [2][3] to lasers and is now widely used thanks to its first order insensitivity to low frequency laser amplitude noise.
The Pound-Drever-Hall technique uses the interference between the reflected field exiting the cavity and the not-resonant sidebands generated by a phase modulation. The sidebands act as a phase reference for the field before its entrance into the cavity. Since the carrier has passed through the cavity it is possible to

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measure the detuning between laser and cavity, thanks to the interference between carrier and sidebands.
The main noise sources for this technique are: the laser noise spectral density at the demodulation frequency (hence the choice of high frequencies for modulation) and the residual amplitude modulation. The former of these effects is significant because on the photodiode we measure the sidebands phase modulation converted into amplitude modulation. To reduce this effect the modulation frequency as already mentioned, is chosen in the range where the laser noise is shot noise limited for the working power ($\nu_m \geq$ 5-10 MHz).
With respect to amplitude modulation the technique is sensitive to this effect, which will result in an error signal offset as we have shown[11]. Minimisation of this effect is obtained through an accurate orientation of the EOM crystal in the experimental apparatus. The technique is insensitive to beam displacements, since an optical misalignment will only influence the quantity of light incident on the photodiode (DC signal), but will not be detectable at the modulation frequency (AC signal). The main drawback of this type of technique for a space application is the power budget. Modulation and demodulation require active components which impact the energetic budget significantly. Also Electro-Optic Modulators are difficult to space qualify and remain a fragile, environment-sensitive component.
We therefore look for a technique which can potentially reach the same performances than Pound-Drever-Hall but which enjoys better space compatibility.

## 3. TILT LOCKING

Tilt Locking is a DC locking technique proposed for the first time by Gray in 1999[4]. In this technique the property of a cavity of expanding the input field in the bases of the TEM Gaussian Modes is used to create a reference beam which will then interfere with the carrier on a split photo-detector. In particular a small misalignment of the input beam on the Fabry-Perot resonator will generate an odd mode of the first order (i.e. $TEM_{10}$ or $TEM_{01}$ depending on the direction of the misalignment). Errors in the adaptation of the beam size to the cavity's intrinsic waist shall result, instead, in an even mode contribution, these too could ideally be used to lock the laser frequency as proposed by Shaddock[5] in his thesis work.
In the case of Tilt Locking it is the $TEM_{01}$ modes that are used as reference beams while the $TEM_{00}$ will be the carrier. If the generated $TEM_{01}$ are not degenerated with the $TEM_{00}$, when the latter is resonant with the Fabry-Perot, the former will be completely (in the ideal case) reflected.
The spatial profiles of the two modes in exam in the one-dimensional case are shown in Fig.1.

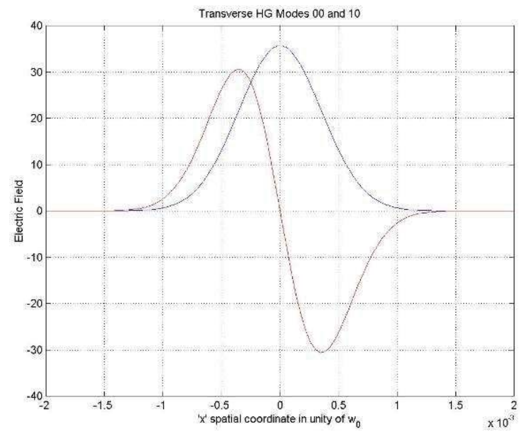


Fig.1. $TEM_{00}$ and $TEM_{10}$: one-dimensional profile.

The interference of these two modes on the two different halves of a split photodiode will produce, through subtraction of the two contributions, the error signal. This interference is of course influenced by the phase difference between the modes, which in its turn depends from both the type of misalignment introduced and the distance of the detector. Additionally the carrier will experiment a phase shift of $\pi/2$ around the resonance which will assure the efficiency of the method. The interference on the two different halves will therefore vary with the laser frequency.
As well known for auto-alignment systems while angular displacements induce phase shifts in the near field condition, shifts will only cause a phase shift in the far field condition. If the photodiode is close to the output mirror, as is advisable to reduce beam jitter noise effect, we are in the near field condition; it will therefore be the tilt of the beam that shall give an error signal. This technique is very attractive for space applications given its lack of active components (EOM and demodulation electronics) and is analogous in this to the more complex, polarisation based, Hänsch-Couillaud[6] and orthogonal mode analysis (Harvey[7]).
Since the error signal is obtained through a misalignment we expect the lock point and the error signal line-shape to be very sensitive to all beam jitter motions. This could indeed be a problem in payloads which are subjected to vibration as is the case in space experiments. To reduce as much as possible the beam jitter noise the solution found by Shaddock et al.[5] was the so called Double Pass Tilt Locking.

## 4. TILT LOCKING DOUBLE PASS

Since Tilt locking is sensitive to optical misalignments, it is important to reduce beam jitter noise as much as possible.
The obvious solution is to shorten the path between the laser source and the entrance mirror of the cavity and

reduce the number of optics crossed by the beam, but this is often not practicable once we come to the experimental application. An elegant solution is obtained by using the Fabry-Perot resonator as a mode cleaner on a first passage and subsequently re-injecting the beam into the cavity with a slight misalignment giving the Tilt. The two beams, being counter-propagating, don't interfere with one another; it is therefore possible to use the reflection off the cavity's second pass entrance mirror to obtain an error signal. The tilt is obtained by misaligning the zero incidence $R_{max}$ mirror that is used to re-inject the beam into the cavity. In this configuration only the beam jitter of the cavity with respect to the re-injection mirror (and eventually lenses and other optics interposed) and the photo-detector is to be taken into account.

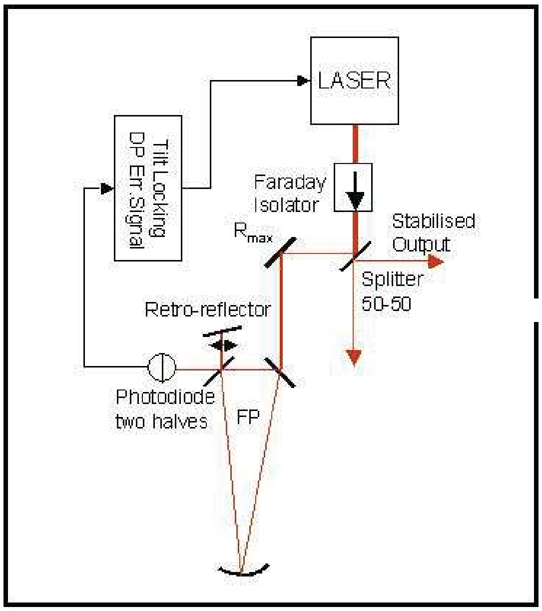


Fig.2. Tilt Locking: Double Pass configuration with cat's eye.

In order to minimise the amount of beam displacement due to the use of a single re-injection mirror we use a cat's eye configuration with a lens interposed between the mirror and the cavity. This lens will have a double function: on the one side it will allow the creation of a pure Tilt and on the other it will refocus the beam, better matching the second pass of the beam into the reference.

It has to be observed that the beam transmitted by the cavity is perfectly Gaussian thanks to the mode cleaning effect of the Fabry-Perot resonator. In this configuration we no longer have an error signal when the $TEM_{01}$ is resonant with the FP. Additionally for a loss of the lock the beam will be entirely reflected off the cavity at its first pass. The photodiode, placed as it is on the second pass reflection, won't have to deal with the whole beam power, at each lock loss.

The main problem with this kind of experimental configuration resides in the delay given by the passage of the light into the cavity. As has been shown by Shaddock [5] in his PHD thesis, though, this will limit the lock bandwidth for frequencies higher than the kHz (bandwidth of the reference). Another inconvenient due to this configuration is the unavoidable backscattered light towards the laser head. This can be solved by using a Faraday isolator or an AOM. The Faraday isolator could be problematic, since the presence of a strong magnetic field on board of most satellites would be unadvisable. The use of an AOM shifter would produce a drain on the energetic budget, thus eliminating the main advantage of Tilt Locking. Studies at LZH in Hannover [9] have shown that NPRO Nd:YAG lasers such as the one built for LISA can receive up to 4% of their output light back in the laser cavity without showing appreciable variations in the phase and power noise spectra. A configuration without Faraday isolator could be acceptable, if it scatters back less than 4% of the originally emitted power. Let us now show the dependency of the error signal lineshape and locking point on the optical misalignment errors (shift and detector offset).

### 4.1 Error signal dependency on misalignments

In our calculations a triangular Fabry-Perot cavity with entrance mirror 1 (reflectivity $r_1$ and transmissivity $t_1$), exit mirror 2 ($r_2$ and $t_2$), and end mirror 3 ($r_3$ and $t_3$) is used. The low finesse of this cavity is assumed to be 1,000 and its high finesse 30,000. Let us now choose a reference frame in which the z axis is along the beam propagation, x is parallel and y perpendicular to the optical table. The laser output, after the first passage into the cavity; is a pure $TEM_{00}$ mode: its form is given by

$$E_{00} = T(\omega) \cdot E_{inc}, \qquad (4.1)$$

with

$$T(\omega) = \frac{t_1 t_2 r_3 \cdot e^{2\pi i \omega \tau}}{1 - r_1 r_2 r_3 \cdot e^{2\pi i \omega \tau}}. \qquad (4.2)$$

Since $T$ isn't a function of the spatial coordinate but only of the detuning we can multiply the expression of the Error Signal for the square modulus of the transmission of the cavity to obtain the double pass Error Signal. This term is a function of the coupling in the cavity $\zeta_0$, the laser frequency detuning $\delta\omega$ and the cavity pole frequency $f_P$ and is, obviously, real:

$$|T(\delta\omega)|^2 = \frac{1 - \zeta_0^2}{1 + \frac{\delta\omega^2}{f_P^2}}. \qquad (4.3)$$

This purely gaussian beam (Eq.4.1) will be injected into the FP with an angular displacement (Tilt) of an angle α and a linear displacement on the entrance mirror (Shift) called *a*. Both of these effects will cause a phase term to appear in the electromagnetic field expression. In particular the tilt, for small values of α, shall give:

$$\Phi_{tilt} = \frac{2\pi\alpha x}{\lambda}, \tag{4.4}$$

whereas in the same conditions the shift will result in:

$$\Phi_{shift} = \frac{2\pi \cdot iax}{\pi w_0^2} + \frac{2\pi}{\lambda}\frac{z^2}{z_R^2}\frac{ax}{\sqrt{z^2 + z_R^2}} \tag{4.5}$$

In these expressions $w_0$ is the beam's waist and $z_R$ is the Rayleigh parameter. Having considered that:

$$E = E_{00} \cdot e^{i\Phi_{tot}} \tag{4.6}$$

and having developed in series to the first order, for small values of *a* and α, we have the following expressions of the electromagnetic field, for a tilt:

$$E_{tilt} = E_{00} + i\frac{\pi\alpha \cdot w(z)}{\sqrt{2}\lambda} \cdot E_{10}, \tag{4.7}$$

and for a shift:

$$E_{shiftt} = E_{00} + \left[\frac{i\pi a \cdot w(z)}{\sqrt{2}\lambda R(z)} - \frac{a}{\sqrt{2}w(z)}\right] \cdot E_{10}. \tag{4.8}$$

If we develop the total field in series to the first order, for small values of a and α, we have the following expressions of the electromagnetic field, for a tilt:

$$E_{tilt} = E_{00} + i\frac{\pi\alpha \cdot w(z)}{\sqrt{2}\lambda} \cdot E_{10}, \tag{4.9}$$

and for a shift:

$$E_{shiftt} = E_{00} + \left[\frac{i\pi a \cdot w(z)}{\sqrt{2}\lambda R(z)} - \frac{a}{\sqrt{2}w(z)}\right] \cdot E_{10}. \tag{4.10}$$

R(z) is the beam curvature in function of the coordinate z, $E_{00}$ and $E_{10}$ are the electromagnetic fields of the TEM modes of the zero and first order respectively. Along the x direction, the one of the tilt, the first order mode can be written with respect to the zero order one as:

$$E_{10} = E_{00} \cdot \frac{\sqrt{2}x}{w(z)} \cdot e^{-i \cdot atg\left(\frac{z}{z_R}\right)} \tag{4.11}$$

$atg\left(\frac{z}{z_R}\right)$ is known as the Gouy phase shift. Now the reflected field shall have the following form:

$$E_{Refl} = R(\omega) \cdot E_{00} + A(a, \alpha, z) \cdot R(\omega_{10}) \cdot E_{10}, \tag{4.12}$$

where $\omega_{10}$ is 2π the (1,0) mode frequency, of a cavity of semi-perimeter L and end mirror radius of curvature R, given by:

$$\omega_{10} = \omega \cdot \left(1 + a\cos\left(\sqrt{1 - \frac{L}{R}}\right)\right) = \omega(1 + \beta) \cdot \tag{4.13}$$

R(ω) and R($\omega_{10}$) are complex reflectivities of the reference cavity at the given frequencies, which can be written as a function of the coupling $\zeta_0$, the detuning $\delta\omega$ and the cavity pole frequency $f_P$.
Integrating the power ($E_{Refl}^* E_{Refl}$) incident on the photodiode in function of the x coordinate on the two halves of a photodiode shifted spatially of $\varepsilon$:

$$P_L = \int_{-\infty}^{\varepsilon} P(x)dx \qquad P_R = \int_{\varepsilon}^{\infty} P(x)dx \tag{4.14}$$

we obtain for the error signal:

$$ES = -\int_{-\infty}^{\varepsilon} P(x)dx + \int_{\varepsilon}^{\infty} P(x)dx \cdot \tag{4.15}$$

If we consider $\Phi_{Gouy}$=0, stop at the first order in $\varepsilon$, and write the reflectivity as a function of the coupling, the detuning and the pole frequency of the cavity we can write the error signal analytically[8] as:

$$ES \cong \frac{2}{\pi w_{inc}^3}\left[\begin{array}{l} -\frac{\alpha\pi w_{inc}}{\lambda}\cdot\left((\zeta_0 - 1)\cdot\beta\frac{\delta\omega}{f_P} + \ldots\right) \\ -\frac{a}{w_{inc}}\left(\zeta_0^2 + \ldots\right) \\ +\frac{2}{\pi}\frac{1}{w_{inc}^2}\varepsilon\left(\zeta_0^2 + \ldots\right) \end{array}\right] \cdot \frac{1 - \zeta_0^2}{1 + \frac{\delta\omega^2}{f_P^2}} \tag{4.16}$$

where with δω we indicate the detuning of the laser with respect to the cavity and with $w_{inc}$ the waist on the input mirror. This formula shows us that the error signal depends under all conditions on the beam misalignment, both linear and angular, and from the detector's alignment.

The Gouy shift phase term has been supposed to be zero, this implies that the previous calculation is only true in the near field condition. In the far field condition it is the linear displacement that will give an error signal, the angular displacement will conversely be linked to an offset effect. This can be understood if we consider that in the far field condition the Gouy phase shift value is close to $\pi/2$. But a phase shift of $\pi/2$ does no more than turn the real part into an imaginary part and this results in the parameters $\alpha$ and $a$ exchanging multiplying constants.

## 4.2 Tilt locking in space: monolithic reference

Let us, first of all, determine which properties a FP resonator would require for Tilt Locking to be compatible with a LISA like experiment. As shown in Eq.4.16 Tilt locking is not sensitive to misalignments as long as the cavity is perfectly coupled ($\zeta_0$=0). This however doesn't hold true in the case where the entrance and exit mirrors of the cavity don't have equal reflectivities. Supposing the misalignments to be of the order of the μm which is compatible with a monolithic assembly (be it silicate bound or optically contacted), we can proceed to estimate the maximum reflectivity difference tolerable for the application of Tilt locking in LISA. For a cavity with a FSR of 1GHz, Finesse $10^5$ ($10^4$) and waist 400μm, a photodiode with a diameter of 1mm and a dead area of 40μm we have that the superior limit for the parameter $r_2{\cdot}r_3{-}r_1$ is of 50ppm, which can be realised in LMA in Lyon [10]. If this condition is satified, our MATLAB simulation of Tilt Locking predicts locking point frequency shifts of 0.8Hz (8Hz), which are compatible not only with the LISA specifications but with many other space applications. This frequency offset value is obtained for the worst case scenario for the offsets, where we have 1μm of detector offset and no beam offset. If we consider a cavity with re-injection prism in the presence of which the beam and detector offsets shall have the same sign the presence of both effects leads to a partial cancellation of the offset as shown by Eq.4.16. It also has to be observed that for a given beam offset, a value of the Gouy shift can always be chosen that shall allow us to obtain an odd error signal line-shape. In fact, the quadrature odd signal is given by the presence of a tilt only in the near field configuration, in the far field it shall be given by the shift. Analogously the even error signal component deforming the line-shape shall be due to the tilt (shift) in the far (near field) configuration respectively. Comforted by this observation let us now consider the experimental set-up needed for Tilt Locking. This would have ideally to be a monolithic assembly grouping the cavity, the re-injection mirror and the detector. A very high Finesse ($10^6$) reference cavity consisting of a silicate bound ULE spacer, dihedron, silica mirrors, injection prisms and detectors, like the one shown in Fig.3, should provide an adequate frequency reference.

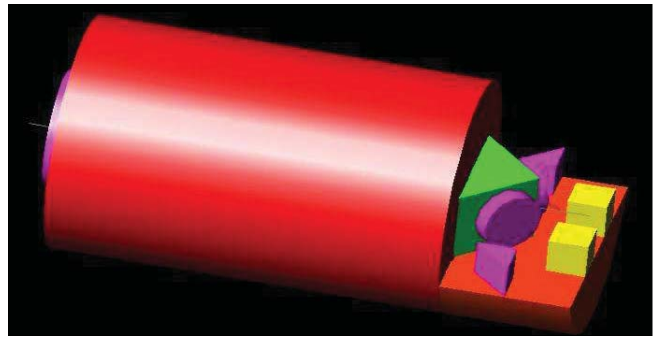

Fig.3. Cavity for Tilt Locking prototype

## 5. CONCLUSIONS

As shown the Tilt Locking technique is a promising technique to be used in space applications once an adequate reference cavity has been built. Tilt Locking is potentially applicable to stabilize the lasers required for the spatial project of detecting gravitational waves LISA.

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