Demonstrating a modal approach to paraxial light propagation for photonics education

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Abstract: Propagation using numerical approaches is a textbook standard, yet suffers from lack of physical insight. We outline a novel modal approach to propagation, demonstrating the ease and physical insight necessary for teaching and to facilitate understanding in photonics courses.

1. Introduction

Traditional propagation calculations in photonics textbooks and courses pose a daunting task for beginners. The angular spectrum method is a complex numerical calculation that requires knowledge of 2D Fast-Fourier Transforms (FFTs) and their inverses, additionally it lacks physical insight into the nature of propagation making it relatively complicated for many students to fully grasp. The need to develop an approach to model this fundamental calculation in an easy-to-understand-and-apply manner is crucial to the growth of educational resources in photonics. We, therefore, developed an intuitive and instructive method to propagate arbitrary optical fields from a modal perspective allowing for a clear, fast and comprehensive calculation, illustrated in Fig. 1. We decompose an initial field at the plane $z = 0$ into an appropriate basis with a known z-dependent propagation function. Each basis element in the decomposition can be propagated analytically, and therefore, so too can the entire initial field which may not have any known analytical propagation rule. To illustrate the ease of implementation and accuracy of the approach, we compare it to the numerical angular spectrum approach, showing excellent agreement, and then validate the method by experiment. We believe that this approach is a powerful and intuitive resource for educational institutions specialising in optics and photonics.

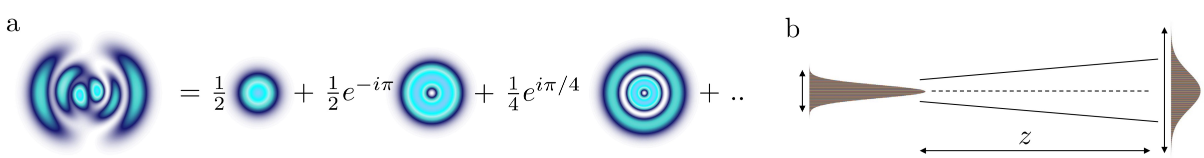

Fig. 1. (a) An arbitrary exotic field, shown on the LHS, can be decomposed into a sum of eigenmodes with complex weightings, on the RHS. (b) Each eigenmode on the RHS has an analytical z-dependent propagation function, whose sum returns the propagation of the arbitrary mode.

2. Modal propagation

Propagation is an abstract concept for most students. Students know and understand that light travels, however comprehending propagation dynamics presents challenges particularly for undergraduate students. Although numerical approaches provide accuracy they add to these challenges due to their insufficient physical insight. Modal propagation is a straightforward approach to propagation: one will first perform a modal expansion of an initial arbitrary field, $u(x,y,z=0)$, into an orthonormal basis $\psi_i(x,y,z=0)$ where $u(x,y,z=0) = \sum_i c_i \psi_i(x,y,z=0)$. Following this, to find the unknown coefficients c_i one would perform a modal decomposition, which can be done numerically or optically [1]. The choice of basis must be restricted to those with known propagation rules, i.e., $\psi_i(x,y,z=0) \xrightarrow{z} \psi_i(x,y,z)$, then since each basis element has a known z dependence, it is now possible to find the propagation of the initial field by

$$u(x,y,z) = \sum_i c_i \psi_i(x,y,z)\,. \tag{1}$$

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There are at least two advantages to such an expansion: (1) the propagation becomes computationally simple since one need only perform a modal decomposition once, on the initial field, and thereafter only analytical propagation of each basis element is performed; (2) the propagation becomes more intuitive. In pedagogy this is imperative as the modal propagation approach provides an analytical propagation method for arbitrary optical fields, thereby circumventing the computational complexities arising from numerical methods i.e. numerical artefacts.

3. Ease of instruction in teaching environments

Analytical calculations are simple for students to perform as there exists an intrinsic understanding arising from the ability to solve the calculation "by-hand". The concept of modal propagation is both easy to teach and to follow due to its analytical nature. Students are able to understand how each element of the basis propagates without experiencing numerical issues while educators can easily demonstrate the method by a series of analytical expressions following the initial numerical or optical modal decomposition, in three simple steps. We outline the steps here for ease of instruction in classroom settings: (1) Choose an orthonormal basis with known and analytical z-dependence, e.g. the Laguerre-Gaussian (LG) basis, (2) Find the coefficients by performing a modal decomposition as such:

$$c_i = \iint u(x,y,0)\mathrm{LG}_{p,\ell}(r,\phi,z=0)\,dA\,. \tag{2}$$

each element of the LG basis has a known z dependence given by:

$$\mathrm{LG}_{p,\ell}(r,\phi,z) = \sqrt{I_0}\mathrm{L}_p^{\ell}\left(\frac{2r^2}{w^2(z)}\right)\left(\frac{\sqrt{2}r}{w(z)}\right)^{|\ell|} \times \exp\left(-\frac{r^2}{w^2(z)}\right)\exp(i\eta(r,\phi,z))\,, \tag{3}$$

where

$$\eta(r,\phi,z) = -kz - \frac{kr^2}{2R(z)} - \ell\phi + (2p+|\ell|+1)\arctan\left(\frac{z}{z_R}\right),\; I_0 = 2\mu_0 c\frac{2Pp!}{\pi w^2(z)(p+|\ell|)!)}\,,$$

(3) Propagate the initial field by summing over all analytically propagated basis elements and their corresponding coefficients:

$$u(x,y,z) = \sum_{p,\ell} c_{p,\ell}\mathrm{LG}_{p,\ell}(r,\phi,z)\,, \tag{4}$$

4. Experimental and numerical validation

Validating the concept experimentally we chose an easy to follow method that both educators and students can replicate in any optics and photonics laboratory. The experiment involved passing a visible laser beam ($\lambda = 633$ nm) through a polariser orientated for horizontal polarisation before being expanded by an objective lens and then collimated by a second lens to overfill the screen of a Spatial Light Modulator (SLM). The SLM was encoded with an appropriate computer generated hologram to create the desired field to be tested, requiring complex amplitude modulation [2, 3]. The desired mode was imaged by lenses $f_2 = f_3$, with an aperture at the Fourier plane used to remove unwanted diffraction orders. A camera was used to measure the beam profiles from the image plane ($z = 0$) as a function of z by moving the camera on a rail. The second moment width of the beam at each position was calculated from the captured images. To measure the far-field and to observe the beams passing through their waist planes, we employed a digital lens of focal length f programmed on the SLM rather than a physical lens. To quantify the agreement, we measure beam images from $z = 0$ to $z = 400$ mm and calculate the second moment beam radius in the two orthogonal axes, with the results for the flat-top and exotic beams shown in Figure 2. We overlay, on the modal propagation approach, the traditional angular spectrum (AS) approach. It is evident that there is perfect agreement between both calculations (AS and Modal) and the measured (Exp) results [4].

5. Conclusion

The modal propagation approach is useful for all educational environments as it has the advantages of being analytical, computationally simple and offers physical insight into the nature of propagation dynamics. Here we have outlined the approach, used it to offer an intuitive understanding of paraxial light propagation, validated it against the traditional angular spectrum method and confirmed it experimentally. We see this approach as a powerful and intuitive tool to be used in both teaching and research laboratories alike.

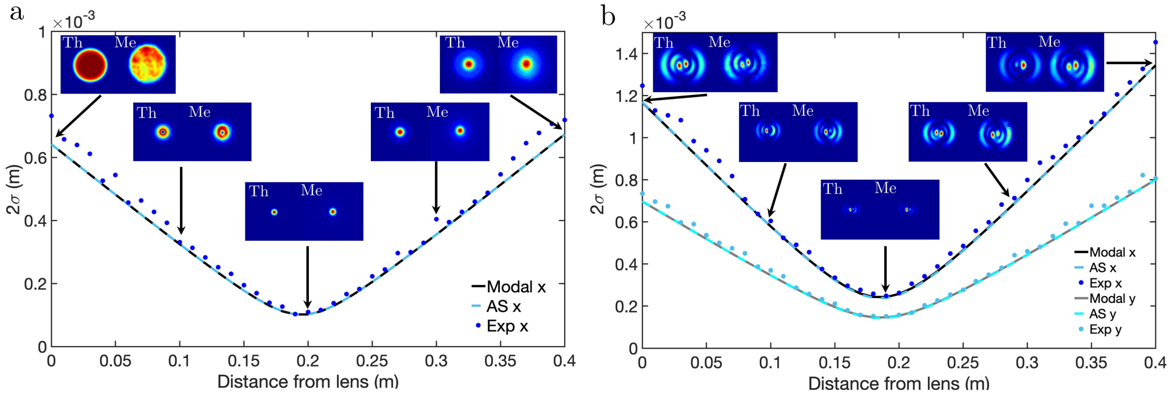

Fig. 2. Second moment beam widths in the $x-$ and $y-$axis as a function of propagation distance (z) for the (a) flat-top and (b) exotic beam, comparing experimentally measured widths (Exp) to those calculated by the angular spectrum (AS) and modal (Modal) approaches. The insets show the measured intensities (Me) and the theoretical (Th) intensities from the modal approach.

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